Lesson 19: Latitude and Longitude

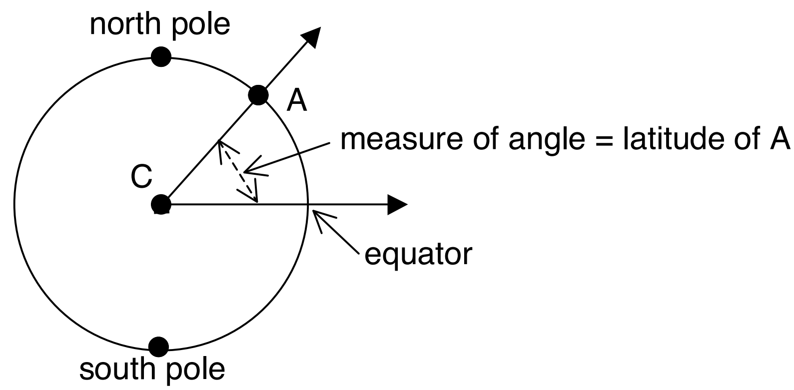
Every point on a globe (with the exception of the north and south poles) has *latitude* and *longitude coordinates*. (The north and south poles have latitude coordinates, but no longitude coordinates.)

Every point on the equator has latitude coordinate 0º. The north pole has latitude +90º or 90º N. ("N" stands for "north".) Points in the *northern hemisphere* (on the same side of the equator as the north pole) have latitude coordinates ranging from 0º to 90º N. The south pole has latitude –90º or 90º S. ("S" stands for "south".) Points in the *southern hemisphere* (on the same side of the equator as the south pole) have latitude coordinates ranging from 0º to 90º S.

The latitude of a point A on a globe can be thought of as representing the measure of the following angle.

- The angle lies in a plane that contains the center C of the globe, the north and south poles and the point A.
- The vertex of the angle is at the center C of the globe.
- One side of the angle starts at C and runs through the equator.
- The other side of the angle starts at C and runs through the point A.

The measure of this angle is the latitude of the point A. The latitude of A has an "N" or "S" attached to it depending on whether the point A lies north or south of the equator.



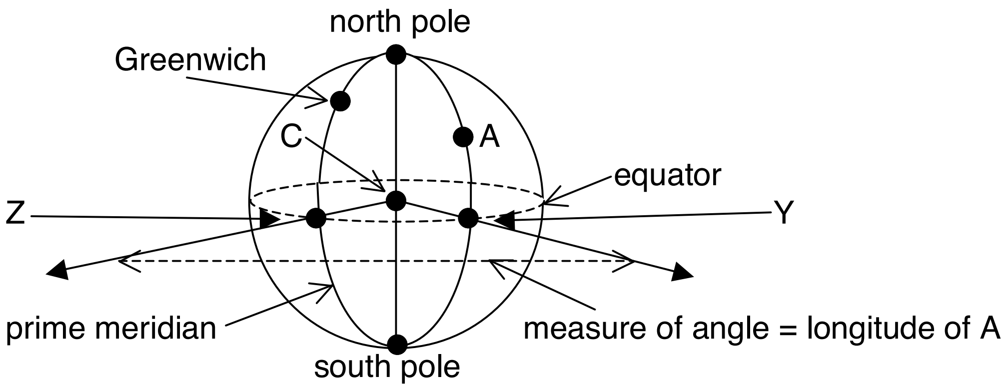
There is an arbitrary division of the Earth into the *eastern* and *western hemispheres.* The great circle that creates this division is the one that passes through the north and south poles and the old Royal Astronomical Observatory in Greenwich, England. This great circle is the union of two meridians of longitude (semicircles joining the north and south poles). One of these meridians joins the north pole to the south pole and passes through Greenwich, England; this meridian is called the *prime* *meridian.* The other meridian joins the north pole to the south pole on the opposite side of the globe from Greenwich, England; this meridian is called the *international date line.* (The international date line passes through the point on the globe that is antipodal to Greenwich, England.) This great circle (which is the union of the prime meridian and the international date line) divides the globe into two hemispheres. The hemisphere which contains North and South America is called the *western hemisphere;* and the hemisphere which contains Asia, Australia, most of Europe and most of Africa is called the *eastern hemisphere*.

Each point that lies on the prime meridian including the old Royal Astronomical Observatory in Greenwich, England (excluding the north and south poles) has longitude coordinate 0º. (Since the north and south poles lie on every meridian, they don't have well defined longitude coordinates.) The points of the eastern hemisphere have longitude coordinates that range from 0º to 180º E, and the points of the western hemisphere have longitude coordinates that range from 0º to 180º W. ("E" stands for "east" and "W" for "west".) Each point on the international date line has two longitude coordinates: 180º E and 180º W. Hence, when longitude coordinates are being considered, 180º E = 180º W. (Why does it make sense for every point with longitude coordinate 180º E to also have longitude coordinate 180º W?)

The longitude of a point A on a globe can be thought of as representing the measure of the following angle.

- The angle lies in a plane that contains the equator and the center C of the globe. (This plane is perpendicular to the line that passes through the north and south poles.)
- The vertex of the angle is at the center C of the globe.
- Let Z be the point where the prime meridian crosses the equator. (Z has latitude 0º and longitude 0º.) One side of the angle starts at C and runs through the point Z.
- Let Y be the point where the meridian that contains A crosses the equator. (Y has latitude 0º and the same longitude as A.) The other side of the angle starts at C and runs through the point Y.

The measure of this angle is the longitude of the point A. The longitude of A has an "E" or "W" attached to it depending on whether the point A lies in the eastern or western hemisphere.



Parts of a degree of longitude or latitude can be indicated in decimal form. For example, the UWM campus has latitude 43.08º N and longitude 87.88º W. Alternatively, parts of a degree can be expressed in *minutes* and *seconds.*

$$1 \text{ minute } = \frac{1}{60}\text{degree.}$$

$$1 \text{ second } = \frac{1}{60} \text{ minute } = \frac{1}{3600} \text{ degree.}$$

When longitude and latitude are expressed in degrees, minutes and seconds, the number of degrees is followed by the symbol º , the number of minutes is followed by the symbol ´ , and the number of seconds is followed by the symbol ´´ . For example, 22º54´17´´ means 22 degrees, 54 minutes and 17 seconds. (Since there are 60 minutes in a degree and 60 seconds in a minute, neither the number of minutes nor the number of seconds can ever exceed 60.) It is easy to convert longitude or latitude expressed in degrees, minutes and seconds to decimal form. For example,

$$22^\circ 54' 17'' = 22 + \frac{54}{60} + \frac{17}{3600} \text{ degrees } =$$

$$22 + .9 + .0047222\ldots \text{ degrees } = 22.9047222\ldots^\circ$$

We will leave the problem of converting longitude and latitude expressed in decimal form into degrees, minutes and seconds for a Homework Problem. Since a difference in latitude or longitude of one second is quite small (less than 1/50 of a mile), then longitudes and latitudes of geographical locations like cities are usually expressed only in degrees and minutes, not seconds.

Activity 1. Groups should discuss and solve the following problem and report their results to the class. Let A, A´, B, B´, C, C´, D, D´ and E, E´ be five pairs of points of the glove with the following latitude and longitude coordinates.

A = 28° S and 51° W	A´ = 13° N and 51° W
B = 36° N and 171° E	B´ = 36° N and 151° W
C = 51° S and 110° E	C´ = 86° S and 70° W
D = 47°S and 129° W	D´ = 47° S and 91° W
E = 36° S and 18° W	E´ = 36° S and 23° E

Let a be the distance along the surface of the globe between the points A and A´.
Let b be the distance along the surface of the globe between the points B and B´.
Let c be the distance along the surface of the globe between the points C and C´.
Let d be the distance along the surface of the globe between the points D and D´.
Let e be the distance along the surface of the globe between the points E and E´.
Fill in the blanks in the following long inequality with the letters a, b, c, d and e to form a correct statement about the relative sizes of a, b, c, d and e.

________ < ________ < ________ < ________ < ________

Activity 2. Groups should discuss and solve the following problem and report their results to the class. Find a method for converting the latitude and longitude coordinates of a point on a globe to the latitude and longitude coordinates of its antipodal point. Apply your method to find the precise latitute and longitude coordinates of the five points on the Earth that are antipodal to the following five cities.

Milwaukee, Wisconsin	43°5´ N and 87°53´ W
Johannesburg, South Africa	26°12´ S and 28°4´ E
Djakarta, Indonesia	6°11´ S and 106°50´ E
Leningrad, Russia	59°56´ N and 30°40´ E
Papeete, Tahiti	17°40´ S and 149°30´ W

Activity 3. The class should discuss the following question.

A bear walks one mile south, one mile west and one mile north and arrives back at the point where he started. What color is the bear?

Activity 4. The class should discuss and solve the following problem. Suppose that a polar bear starts at the north pole and walks and swims south along the prime meridian all the way to the equator. Then it turns west and walks and swims along the equator until it reaches a longitude of 90º W. Finally it turns north and walks and swims along the 90º W meridian of longitude all the way back to the North Pole. Consider the triangle on the Earth's surface traced by the bear's walk. What is the sum of the measures of the angles of this triangle?

Recall that if A and B are two non-antipodal points on a sphere S, then the *geodesic* joining two points A and B on S is the shorter arc of the great circle on S that passes through A and B. The geodesic joining A to B is the path of the "tight string" stretched between A and B. It is the shortest path from A to B.

Suppose A, B and C are three points on a sphere S such that A, B and C don't all lie on a single great circle. Then the *spherical triangle* on S with vertices A, B and C is the union of the three geodesics on joining A to B, A to C and B to C.

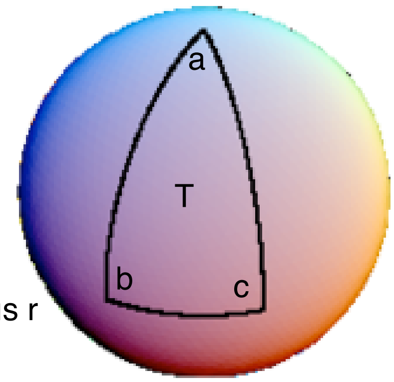
Activity 5. The class as a whole should consider the following question. Suppose that T is a spherical triangle on a sphere S. What general statement, if any, can be made about the sum of the measures of the angles of this spherical triangle. As an aid to considering this question, students should create spherical triangles on globes by stretching pieces of string between three points on a globe. Consider a variety of different spherical triangles.

For spherical triangles, there is a surprising relationship between the sum of the measures of the angles of the triangle and its area. There can be no relationship of this type for planar triangles because the angle sum of a planar triangle is always 180º regardless of how big or small the triangle is. However, for a spherical triangle, the area is proportional to the amount that the angle sum exceeds 180º! This relationship is known by two names – *Harriot's Theorem* and *Girard's Theorem* – after the two mathematicians who independently discovered it – Thomas Harriot (1560-1621) and Albert Girard (1595-1632).[1] We now state the theorem of Harriot and Girard.

[1] Thomas Harriot and Albert Girard led interesting lives. For instance, Harriot was a mathematician, astronomer, navigator and linguist who served Sir Walter Raleigh as scientific advisor, ship designer and accountant and accompanied Raleigh on an expedition from England to Virginia in 1585-86 where he learned the Algonquin language. Harriot used an early telescope for astronomical observations before Galileo became famous for such observations. Biographies of Harriot and Girard and can be found on the internet.

The Theorem of Harriot and Girard. Let T be a triangle on a sphere S of radius r. (The sides of T are geodesics on S.) Let a, b and c be the measures of the angles of T, let S = a + b + c be the sum of the measures of the angles of T, and let A be the area of T. Then

$$A = \pi r^2 \left(\frac{S - 180^\circ}{180^\circ} \right)$$



Sphere of radius r

Example. Let S be a sphere of radius r. Suppose T is a spherical triangle on S with its upper vertex at the north pole and its lower edge along the equator. Then $b = c = 90^\circ$. In this case $S = a + 90^\circ + 90^\circ = a + 180^\circ$. Therefore, $S - 180^\circ = a$. Hence, the Theorem of Harriot and Girard tells us that

$$A = \pi r^2 \left(\frac{a}{180^\circ} \right).$$

In particular, if $a = 90^\circ$, then

$$A = \pi r^2 \left(\frac{90^\circ}{180^\circ} \right) = \left(\tfrac{1}{2} \right) \pi r^2.$$

Observe that if $a = 90^\circ$, then the area A of T is equal to $^1/_4$ of the area of the northern hemisphere of S. Hence, if $a = 90^\circ$, then A is equal to $^1/_8$ of the area of the entire sphere S. Thus, by letting $a = 90^\circ$, we get the following formula for the area of the entire sphere S.

$$\text{Area}(S) = 8A = 8 \left(\tfrac{1}{2} \right) \pi r^2 = 4\pi r^2.$$

Activity 6. The class as a whole should consider the following questions. Suppose S is a sphere of radius r, T is a spherical triangle on S, the angles of T have measures a, b and c, and the area of T is A.

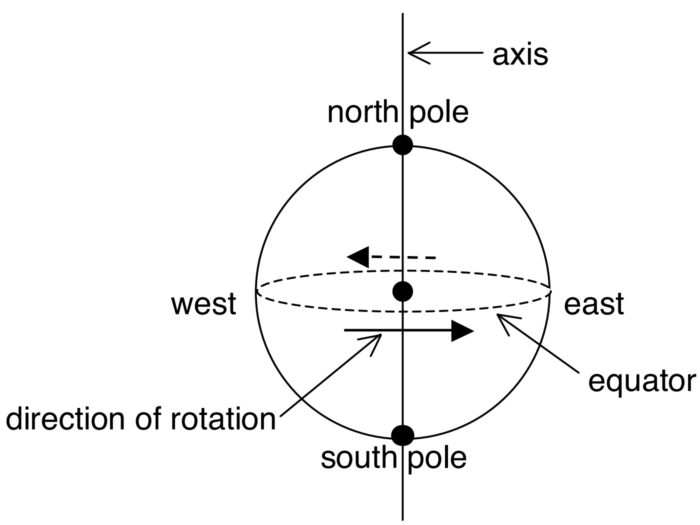
a) Suppose the upper vertex of T is at the north pole of S and the lower edge of T is along the equator of S. Suppose the measure of the angle at the upper vertex of T is a = 120°. What is A? What proportion is A of the entire area of the sphere S?

b) Again suppose the upper vertex of T is at the north pole of S and the lower edge of T is along the equator of S. Suppose the area A of T is $^{1}/_{12}$ of the area of the entire sphere S. What is a?

c) Suppose a = 100°, b = 120° and c = 110°. What is A?

The Earth's *axis* is an imaginary line that passes through the north and south poles and the center of the Earth. When seen from the Sun, the Earth rotates about its axis once every day, or once every 24 hours. This creates the passing of day and night. If the Earth failed to rotate from the point of view of the Sun, then one hemisphere of the Earth would always face the Sun and experience eternal day, while the opposite hemisphere of the Earth would always face away from the Sun and experience eternal night.

Points on the Earth's surface rotate in the direction from west to east. This is consistent with the fact that people on the Earth living to our east experience sunrise and sunset before we do, and the local times on their clocks are ahead of our local times. (Other planets of our solar system rotate about their axes but have different periods of rotation or "days".[2])



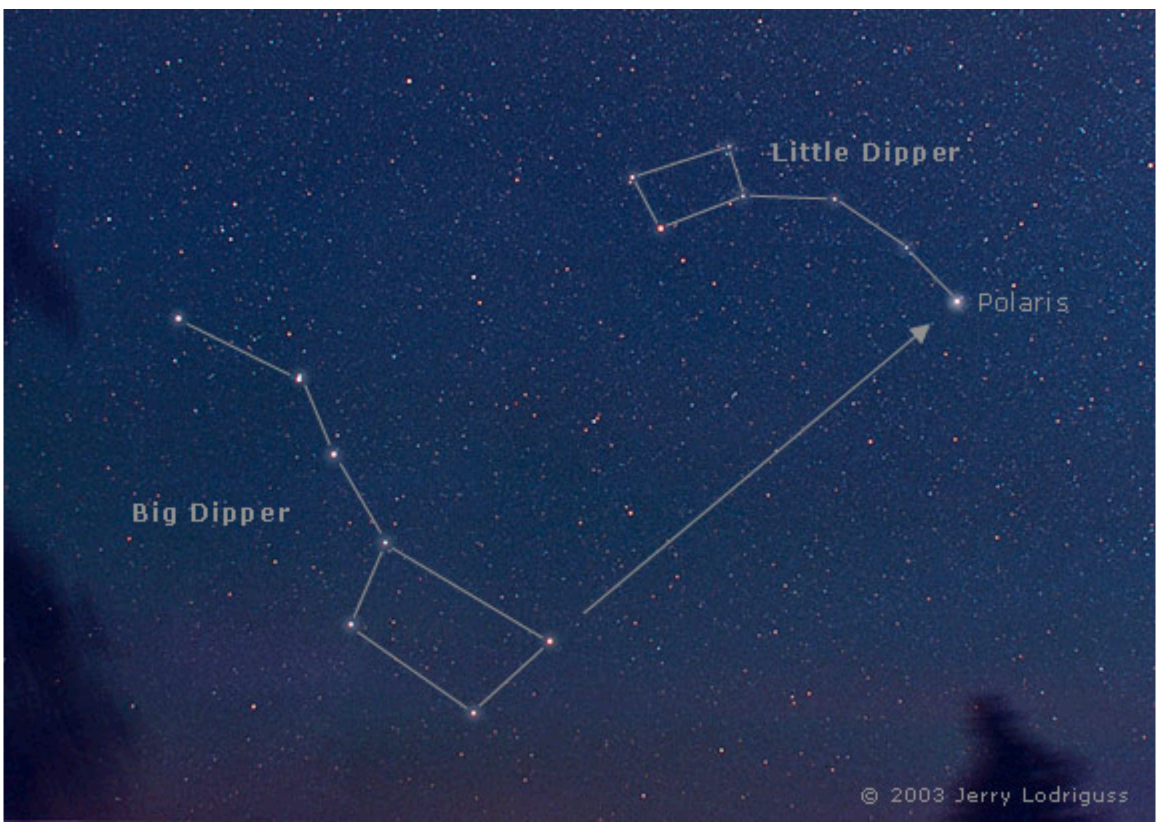
[2]

Planet:	Mercury	Venus	Mars	Jupiter	Saturn	Uranus	Neptune	Pluto	
Length of day: (in Earth days)	58.65		243.02	1.03	.41	.44	.072	.67	6.39

Surprisingly, the giant planets Jupiter and Saturn rotate very quickly on their axes. Their days are less than half of an Earth day.

The Earth's direction of rotation being from west to east is also consistent with the fact that we see the sun moving across the sky in the opposite direction: from east to west. In fact, all astronomical bodies – the Sun, the Moon and the stars rise in the east and set in the west. (The only exception to this rule is that certain stars are so close to the North Star that they never set. These stars move in a counterclockwise direction around the North Star, moving "over" the North Star from east to west and then "under" it from west to east.)

When the Earth's axis extends into space beyond the north pole it points directly at the star called the *North Star* or *Polaris*. For this reason, Polaris does not appear to move from east to west or to rotate in the night sky; it remains fixed. Polaris lies at the end of the "handle" of the constellation known as the *Little Dipper* (which is part of the constellation *Ursa Minor (small bear)*). The standard method for locating Polaris in the night sky is first to find the easily recognizable *Big Dipper* (which is part of the constellation *Ursa Major (large bear)*) in the northern sky. Locate the two stars forming the side of the dipper's cup that doesn't touch its handle. Then follow the line determined by these two stars moving from the bottom to the top and out of the dipper's cup. This first bright star that this line passes through is Polaris. (See the figure on the next page.)



The Earth moves around the sun in an orbit that is an ellipse or oval. This oval is very close to a circle. The point on the Earth's orbit which is closest to the Sun is called the Earth's *perihelion.* This point is 147,090,000 km from the Sun (to the nearest 10,000 km). The point on the Earth's orbit which is farthest from the Sun is called the Earth's *aphelion.* This point is 152,100,000 km from the Sun (to the nearest 10,000 km). Hence, the average distance from the Sun to the Earth in its orbit is 149,600,000 km (to the nearest 10,000 km) or about 92,900,000 miles. The variation of the Earth's distance from the Sun between perihelion and aphelion is about 3% of the average distance. ((152,090,000 – 147,090,000) / 149,600,000 = .033…) The fact that this variation in the Earth's distance from the Sun is a such a small proportion of the average distance justifies our assertion that the Earth's orbit is nearly circular.

We just stated that the average distance from the Sun to the Earth in its orbit is 149,600,000 km (to the nearest 10,000 km). Here is some related information. The diameter of the Sun is 1,392,000 km (to the nearest 1000 km), and the diameter of the Earth is 12,735 km (to the nearest km). This means that if we were to create a scale model of the Sun and Earth in which the distance from the Sun to the Earth were the length of the football field (= 100 yards = 300 feet = 3600 inches), then the scale model of the Sun would have a diameter of

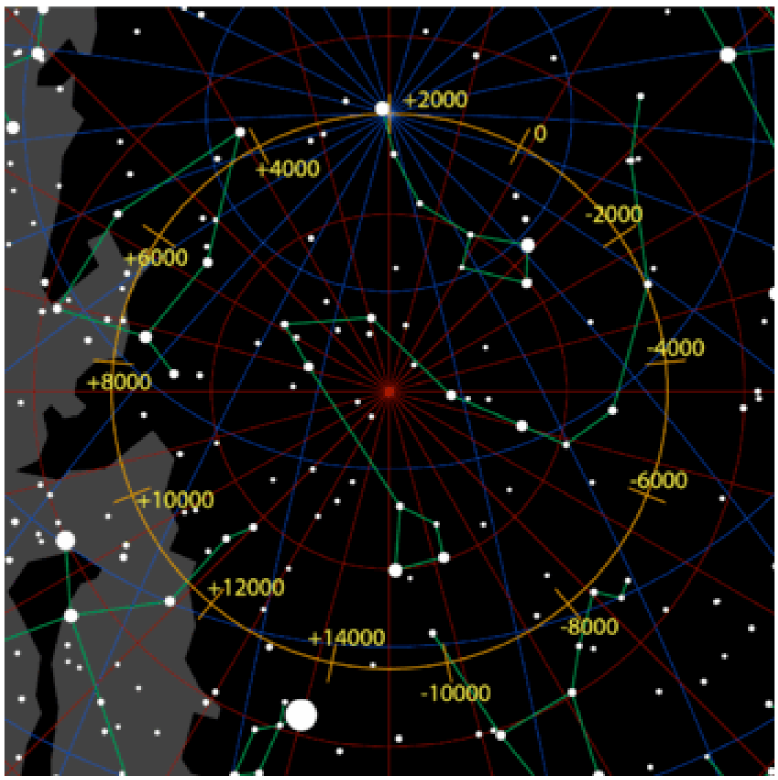
$$\frac{3600}{149{,}600{,}000}\times 1{,}392{,}000 \;=\; 33.5 \text{ inches,}$$

and the scale model of the Earth would have a diameter of

$$\frac{3600}{149{,}600{,}000}\times 12{,}735 \;=\; .3 \text{ inches.}$$

The point of this information is that the distance from the Sun to the Earth is much greater than the diameter of either the Sun or the Earth.

Today the Earth's axis points toward the North Star, but this has not always been the case. As the Earth moves in its orbit around the Sun and rotates daily about its axis, the axis itself "wobbles". More precisely, the axis rotates very slowly in space. This slow rotation of the axis of the Earth is called *precession.* Precession can be seen by observing a quickly spinning gyroscope or top. As the gyroscope spins rapidly around its axis, the axis itself rotates or *precesses*. The axis of the Earth rotates 1° every 72 years. This means that although the Earth's axis points at the North Star today, in several hundred years it will point at a different spot in the sky. The North Star will no longer lie directly north. As the years pass, the Earth's axis will trace out a circle in the sky that has angular radius 23.5°. (This circle is shown in the figure on the next page.) Since the Earth's axis moves 1° every 72 years, then the axis will make one complete revolution around this circle and again point toward the North Star in roughly 360×72 = 25,920 years – say 26,000 years. Thus, in the year 28,010, your descendants will again find that the Earth's axis points at the North Star. The precession of the Earth's axis at the rate of 1° every 72 years was first discovered by the ancient Greek astronomer Hipparchus who lived from 190 BC to 120 BC. When the ancient Egyptians built the great pyramids about 2500 BC, they attempted to align the main corridors of the pyramids along a north-south axis. They could not have accomplished this by pointing the corridors toward the North Star, because 4500 years ago the North Star did not lie to the north. Instead they would have had to point the corridors of the pyramids toward other stars which had been identified by astronomers of their period as lying in a northerly direction. Recently, deviations of the alignment of certain pyramids from true north have been used to estimate the date of construction of these pyramids.[3]



As years go by, the Earth's axis will trace out the circle in the sky shown in the figure above. Today the Earth's axis points toward the North Star which is the bright star shown at the top of the circle near the label "+2000". The North Star is part of the *Little Dipper* constellation which is also easily identified near the top of the circle. The large constellation which stretches through much of the interior of the circle is *Draco* the dragon. When the ancient Egyptians build the pyramids around 2500 BC, they would have looked for north at a point on this circle a little bit below the label "–2000", about half way between the "–2000" mark and the star located where the circle crosses Draco's tail.

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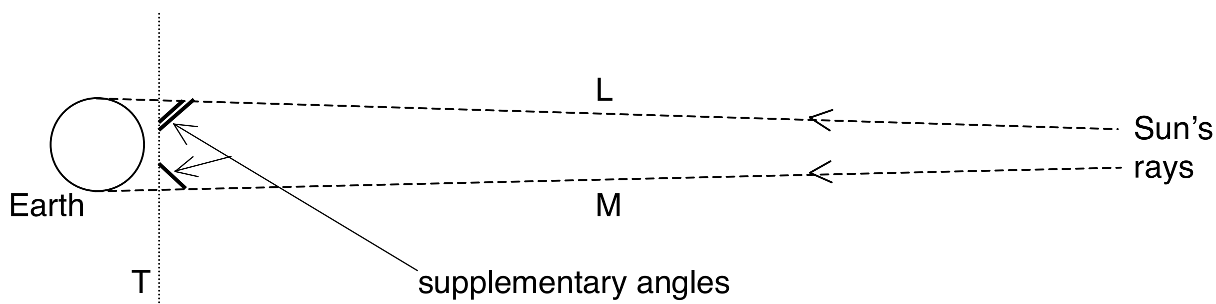
[3] See http://www.americanscientist.org/issues/pub/2004/9/astronomy-and-the-great-pyramid and http://www.nature.com/nature/journal/v408/n6810/full/408320a0.html.

Activity 7. The class should consider the following situation and answer the three questions below. Imagine two light rays L and M emanating from the Sun and arriving at different points on the Earth's surface. Also imagine a line T near the Earth's surface that intersects the lines L and M. (T is a *transversal* to L and M.) Keep in mind that the Earth's diameter is extremely small compared to the distance between the Earth and the Sun. (If the distance between the Earth and the Sun were the length of a football field – 3600 inches, then the diameter of the Earth would be .3 inch.)

a) What can be said about the angle between the rays L and M at their point of origin in the center of the Sun?

b) What can be said about the sum of the measures of the supplementary angles where the rays L and M cross the transversal T?

c) Is it justifiable two say that two light rays traveling from the Sun to different points on the Earth's surface are essentially parallel?



Homework Problem 1. Let A, A´, B, B´, C, C´, D, D´ and E, E´ be five pairs of points on the globe with the following latitude and longitude coordinates.

A = 32º27´ S and 139º15´ E	A´ = 32º27´ S and 70º22´ E
B = 32º27´ N and 41º43´ W	B´ = 32º27´ N and 22º29´ E
C = 39º41´ N and 47º25´ E	C´ = 71º26´ N and 132º35´ W
D = 0º0´ N and 14º51´ W	D´ = 0º0´ N and 85º23´ W
E = 49º34´ S and 170º27 E	E´ = 49º34´ S and 125º21´ W

Let a be the distance along the surface of the globe between the points A and A´.
Let b be the distance along the surface of the globe between the points B and B´.
Let c be the distance along the surface of the globe between the points C and C´.
Let d be the distance along the surface of the globe between the points D and D´.
Let e be the distance along the surface of the globe between the points E and E´.

a) Fill in the blanks in the following long inequality with the letters a, b, c, d and e to form a correct statement about the relative sizes of a, b, c, d and e.

________ < ________ < ________ < ________ < ________

b) Give a written explanation for why each of these four inequality relationships is true.

Homework Problem 2. Write down a procedure for converting the latitude and longitude coordinates of a point on a globe into the latitude and longitude coordinates of the antipodal point. Check that your procedure works by applying it to the five cities listed in Activity 2.

Homework Problem 3. In Activity 2 and Homework Problem 2, you developed a procedure for converting the latitude and longitude coordinates of a point on the globe to the latitude and longitude coordinates of its antipodal point. Apply this procedure to find the precise latitute and longitude coordinates of the five points on the Earth that are antipodal to the following five cities.

Las Palmas, Canary Islands	28º8´ N and 15º27´ W
Brazzaville, Congo	4º16´ S and 15º17´ E
McMurdo Station, Antarctica	77º51´ S and 166º40´ E
Hanoi, Vietnam	21º2´ N and 105º51´ E
Adak Island, Alaska	51º53´ N and 176º39´ W

Homework Problem 4. Devise a method for converting a latitude or longitude coordinate expressed in degrees in decimal form to one expressed in degrees, minutes and seconds. Test your method by converting 22.9047222…° to degrees, minutes and seconds. (According to the calculation on page 75, you should get 22°54´17´´.)

Homework Problem 5. Suppose S is a sphere of radius r, T is a spherical triangle on S, the angles of T have measures a, b and c, and the area of T is A.

a) Suppose the upper vertex of T is at the north pole of S and the lower edge of T is along the equator of S. Suppose a = 144°. What is A? What proportion is A of the entire area of the sphere S?

b) Again suppose the upper vertex of T is at the north pole of S and the lower edge of T is along the equator of S. Suppose the area A of T is $^{9}/_{40}$ of the area of the entire sphere S. What is a?

c) Suppose a = 125°, b = 115° and c = 105°. What is A?