## Lesson 8: Congruent Figures and Rigid Motions

Activity 1. Groups should carry out this activity and report their results to the class. In parts a) through d) of this activity on this and the next page, use a pencil, a ruler and a piece of patty paper to perform the indicated geometric constructions.
a) Construct and label the figure $T_{A, B}($ Sun $)$ and the point $T_{A, B}(P)$.


A

B
b) Construct and label the figure $\mathrm{R}_{\mathrm{C}, \mathrm{a}}(\mathrm{Can})$ and the point $\mathrm{R}_{\mathrm{C}, \mathrm{a}}(\mathrm{Q})$.

c) Construct and label the figure $Z_{L}($ Moon $)$ and the point $Z_{L}(P)$.

d) Construct and label the figure $\mathrm{G}_{\mathrm{A}, \mathrm{B}}($ Star $)$ and the point $\mathrm{G}_{\mathrm{A}, \mathrm{B}}(\mathrm{Q})$.


In Lesson 7 and in Activity 1 of this lesson, we considered the problem of how a particular rigid motion moves a given figure to a new position. We now begin to explore how to invert this process. Given two congruent figures, we will try to discover which rigid motions are capable of moving the first figure to the second. We will take an approach to this problem which is similar to the approach taken by a manufacturing company that wants discover how to produce an item similar to one that was invented and manufactured by one of their competitors. The company would have its engineers take the item apart, study its properties and try to come up with methods that would manufacture an item with these properties. This approach is called reverse engineering. We will use reverse engineering to solve our problem. We will study the relationship between the two given figures and, knowing basic properties of the rigid motions, try to deduce which rigid motion would be capable of moving the first figure to the second.

When a company tries to reverse engineer at item, it often finds that there is more than one way to manufacture an object with the properties of the given item. The same is true in the problem we are facing. Given two congruent figures, there may be more than one rigid motion that is capable of moving the first figure to the second. In other words, our problem may not have a unique solution. There may be more than one solution. This is more likely to be true if the two congruent figures each have some internal symmetry.

Activity 2. Groups should carry out this activity and report their results to the class. In parts a) through d) of this activity on the next two pages, two congruent figure F's labeled "1" and "2" are shown. Discover a rigid motion that moves F number 1 to $F$ number 2. Furthermore:

- If the rigid motion is a translation, locate and label points $A$ and $B$ so that this translation is $T_{A, B}$.
- If the rigid motion is a rotation, locate and label a point $C$ and an oriented angle of measure a so that this rotation is $\mathrm{R}_{\mathrm{C}, a}$.
- If the rigid motion is a reflection, locate and label a line $L$ so that this reflection is $Z_{L}$.
- If this rigid motion is a glide reflection, locate and label points $A$ and $B$ so that this glide reflection is $G_{A, B}$.

Use a pencil, a ruler and patty paper and be as accurate as you can.
Hint. Here is an approach that may help you decide which rigid motion moves $F$ number 1 to $\mathbf{F}$ number 2 in each part of this activity. Draw line segments from several points of $\mathbf{F}$ number 1 to the corresponding points of $\mathbf{F}$ number 2. Examine these line segments while considering the following four questions:

- Does each pair of line segments form the opposite sides of a parallelogram?
- If not, do the perpendicular bisectors of all the line segments pass through the same point?
- If not, do the perpendicular bisectors of all the line segments coincide?
- If not, are the midpoints of all the line segments collinear?
a)

b)

c)

d)


Activity 3. Groups should carry out this activity and report their results to the class. In parts a) through d) of this activity on this page and the next, two congruent figures labeled "1" and "2" are shown. Discover a rigid motion that moves figure 1 to figure 2. Furthermore:

- If the rigid motion is a translation, locate and label points $A$ and $B$ so that this translation is $T_{A, B}$. Also choose and label a point $P$ on figure 1 that is different from points $A$ and $B$; and then locate and label the point $T_{A, B}(P)$.
- If the rigid motion is a rotation, locate and label a point $C$ and an oriented angle of measure a so that this rotation is $\mathrm{R}_{\mathrm{C}, \mathrm{a}}$. Also choose and label a point P on figure $\mathbf{1}$; and then locate and label the point $R_{C, a}(P)$.
- If the rigid motion is a reflection, locate and label a line $L$ so that this reflection is $Z_{L}$. Also choose and label a point P on figure 1; and then locate and label the point $Z_{L}(P)$.
- If this rigid motion is a glide reflection, locate and label points $A$ and $B$ so that this glide reflection is $G_{A, B}$. Also choose and label a point $P$ on figure 1; and then locate and label the point $G_{A, B}(P)$.

Use a pencil, a ruler and patty paper and be as accurate as you can. You may want to make use of the hint given in Activity 2.

## a)


b)

c)

d)

$$
\sum_{S}^{N M}
$$



Activity 4. Groups should carry out this activity and report their results to the class. Based on the methods you used in Activity 1, devise and write down a sequence of steps you can take to solve the following problem.

Given two congruent figures in a plane labeled "1" and "2", find a rigid motion of the plane that moves figure 1 to figure 2.

Here are some guidelines for this activity. (These guidelines are an embellishment of the hint given in Activity 1.) Draw line segments from several points of figure 1 to the corresponding points of figure 2. Examine these line segments while considering questions like:

- Does each pair of line segments form the opposite sides of a parallelogram?
- If not, do the perpendicular bisectors of all the line segments pass through the same point?
- If not, do the perpendicular bisectors of all the line segments coincide?
- If not, are the midpoints of all the line segments collinear?

From your answers to these questions formulate four criteria to decide whether the rigid motion that moves one figure to a congruent figure is a translation, a rotation, a reflection or a glide reflection. The criterion for translation should have the form:

The rigid motion that moves figure 1 to figure 2 is a translation if and only if the line segments joining points of figure 1 to corresponding points of figure 2 have the following property: $\qquad$ .

In the other three criteria, replace the word "translation" by "rotation", "reflection" and "glide reflection" and fill in the blank with the appropriate property.

Furthermore, each criterion should also include a description of how to find appropriate points, lines and/or angles that completely specify the rigid motion. More precisely:

- The criterion for translation should also tell how to find two points $A$ and $B$ so that the translation can be identified as $\mathrm{T}_{\mathrm{A}, \mathrm{B}}$.
- The criterion for rotation should also tell how to find a point $C$ and an oriented angle of measure a so that the rotation can be identified as $\mathrm{R}_{\mathrm{C}, a}$.
- The criterion for reflection should also tell how to find a line $L$ so that the translation can be identified as $Z_{L}$.
- The criterion for glide reflection should also tell how to find two points $A$ and $B$ so that the translation can be identified as $\mathrm{G}_{\mathrm{A}, \mathrm{B}}$.

Homework Problem 1. Each student should write up and hand in a careful and detailed description of the steps and criteria developed by the class in Activity 4 to determine which rigid motion will move one given congruent figure to another.

Homework Problem 2. In parts a) through c) of this problem on this page and the next, two congruent figures labeled " 1 " and " 2 " are shown. Use the criteria you developed in Activity 4 to discover a rigid motion that moves figure 1 to figure 2. Furthermore:

- If the rigid motion is a translation, locate and label points $A$ and $B$ so that this translation is $T_{A, B}$. Also choose and label a point $P$ on figure 1 that is different from points $A$ and $B$; and then locate and label the point $T_{A, B}(P)$.
- If the rigid motion is a rotation, locate and label a point $C$ and an oriented angle of measure $a$ so that this rotation is $\mathrm{R}_{\mathrm{c}, \mathrm{a}}$. Also choose and label a point P on figure $\mathbf{1}$; and then locate and label the point $\mathrm{R}_{\mathrm{C}, a}(\mathrm{P})$.
- If the rigid motion is a reflection, locate and label a line $L$ so that this reflection is $Z_{L}$. Also choose and label a point $P$ on figure 1; and then locate and label the point $Z_{L}(P)$.
- If this rigid motion is a glide reflection, locate and label points $A$ and $B$ so that this glide reflection is $G_{A, B}$. Also choose and label a point $P$ on figure 1; and then locate and label the point $G_{A, B}(P)$.

Use a pencil, a ruler and patty paper and be as accurate as you can.

## a)


b)

c)


Homework Problem 3. In parts a) through d) of this problem on this and the next two pages, two congruent figures labeled "1" and " 2 " are shown. Use the criteria you developed in Activity 4 to discover a rigid motion that moves figure $\mathbf{1}$ to figure 2.
Furthermore:

- If the rigid motion is a translation, locate and label points $A$ and $B$ so that this translation is $T_{A, B}$. Also choose and label a point $P$ on figure 1 that is different from points $A$ and $B$; and then locate and label the point $T_{A, B}(P)$.
- If the rigid motion is a rotation, locate and label a point $C$ and an oriented angle of measure a so that this rotation is $\mathrm{R}_{\mathrm{C}, \mathrm{a}}$. Also choose and label a point P on figure $\mathbf{1}$; and then locate and label the point $R_{C, a}(P)$.
- If the rigid motion is a reflection, locate and label a line $L$ so that this reflection is $Z_{L}$. Also choose and label a point P on figure 1; and then locate and label the point $Z_{L}(P)$.
- If this rigid motion is a glide reflection, locate and label points $A$ and $B$ so that this glide reflection is $G_{A, B}$. Also choose and label a point $P$ on figure 1; and then locate and label the point $G_{A, B}(P)$.

Use a pencil, a ruler and patty paper and be as accurate as you can.

## a)


b)

c)

d)


Homework Problem 4. In parts a) through c) of this problem on this page and the next, two congruent figures labeled "1" and " 2 " are shown, and more than one rigid motion moves figure 1 to figure 2. (This is a result of the fact that the figures have internal symmetry.) Find all the rigid motions that move figure 1 to figure 2. Furthermore:

- If one of the rigid motions is a translation, locate and label points $A$ and $B$ so that this translation is $T_{A, B}$. Also choose and label a point $P_{1}$ on figure 1 that is different from points $A$ and $B$; and then locate and label the point $T_{A, B}\left(P_{1}\right)$.
- If one of the rigid motions is a rotation, locate and label a point $C$ and an oriented angle of measure a so that this rotation is $\mathrm{R}_{\mathrm{c}, a}$. Also choose and label a point $\mathrm{P}_{2}$ on figure 1; and then locate and label the point $R_{C, a}\left(P_{2}\right)$.
- If one of the rigid motions is a reflection, locate and label a line $L$ so that this reflection is $Z_{L}$. Also choose and label a point $P_{3}$ on figure 1 ; and then locate and label the point $Z_{L}\left(P_{3}\right)$.
- If one of the rigid motions is a glide reflection, locate and label points $A$ and $B$ so that this glide reflection is $G_{A, B}$. Also choose and label a point $P$ on figure 1 ; and then locate and label the point $G_{A, B}(P)$.
a)

b)

c)


