Lesson 6: Actually Doing Large Scale Measurements

In this lesson the groups will use apparatuses called *clipboard holders* to perform large-scale measurements of distance and height. The theory behind the process is illustrated in Lesson 5 Homework Problems 1 and 2. The major steps in this process are:

1) Measure a reference height on the object to be measured.

2) Use the clipboard holder to create a diagram consisting of several straight lines. (Detailed instructions for using a clipboard holder are given below.)

3) Possibly tape a piece of paper to the edge of the page with the diagram and/or construct additional straight lines on the diagram.

4) Measure some distances on the diagram.

5) Calculate the desired distance or height by using the *Distance* or *Height Lemma.*

6) Report the result of the calculation to the instructor so that he can record it.

The class will reconvene after all the measurements are completed, and will examine, compare and analyze the results of the measurements and try to explain the variation among them.

To carry out this process, each group will need (in addition to a clipboard holder):

- **pencil and paper** to draw figures and record calculations,
- **tape** to attach a piece of paper to the edge of the page with the diagram,
- a **ruler** to draw straight lines,
- **patty paper** or a **compass** or **T-square** or **right triangle** to construct a perpendicular to a given line,
- a **measuring tape** to measure distances in the created figures, and
- a **pocket calculator** for computations.

Detailed instructions for use of the clipboard holders are on the next page.

Guidelines for Large-Scale Measurements Using Clipboard Holders

1) Set up the clipboard holder. Secure the long and short legs together at the point on the short leg marked by the black tape.

2) Place a blank sheet of paper on the clipboard.

3) Hang a plumb line (a string with a washer on one end) over the piece of paper on the clipboard by hanging the loop on one end of the string over the end of the screw protruding from the back of the clipboard and running the string over the top of the clipboard and down its front.

4) Move the clipboard holder until the surface of the clipboard lines up with the vertical object that contains your reference height. One member of the group should hold his/her eye at the edge of the clipboard that is farther from the object to be measured looking toward this object. The clipboard should be far enough to the object being measured so that from this observer's point of view, the span of the object is *less than the entire length* of the edge of the clipboard closer to the object; however, the clipboard should be close enough to the object being measured that from this observer's point of view, the object spans a *significant portion* of the edge of the clipboard closer to the object.

5) One member of the group places his/her forehead on the edge of the clipboard holder so that the edge of the clipboard is directly in front of one of his/her eyes and he/she is looking at the object to be measured. **From this point on, this person should not move his/her head!** Another member of the group should hold the clipboard holder steady.

6) For each sighting of a point on the object to be measured (its bottom or top or a reference height), make two small marks: one near the edge of the paper nearest your eye, and the other near the opposite edge of the paper. **When you make a mark, the pencil tip should exactly block your view of the object you are sighting.**

7) Another member of the group should make two small marks along the plumb line: one near the top edge of the paper and the other near the bottom edge.

8) Remove the paper from the clipboard and use a ruler to draw a straight line through each pair of marks made for a sighting. Also draw a straight line through the pair of marks made along the plumb line.

9) For your distance measurement only, tape a piece of paper to the edge of the page that was nearest your eye, extend the two non-plumb lines onto the attached piece of paper until they cross – creating a triangle, and construct a line through this crossing point that is perpendicular to the plumb line.

10) Make the necessary measurements on your paper to be able to calculate the desired height or distance.

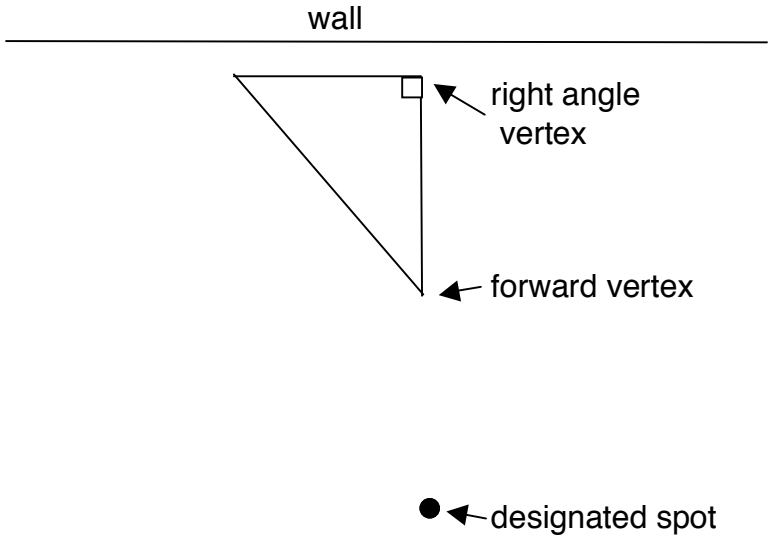
11) Use the *Height* or *Distance Lemma* to calculate the desired height or distance from these measurements.

12) Report the result of the calculation to the instructor.

One of the large-scale distance measuring activities that you will be asked to perform is to find the distance from a designated point to a wall. As part of this activity, you will need to find a point along the wall that is closest to the designated point. We now describe a method for finding this point. This method is simply a practical application of a technique for finding a point on a line that is closest to a designated point not on the line.

Method for Finding a Point on a Wall
Closest to a Designated Point not on the Wall

This method requires a group of four people. First make a 3 foot-by-4 foot-by-5 foot right triangle out of string with washers at the vertices. One member of the group then stands at the designated spot. The other three members of the group hold the washers at the vertices of the string triangle and pull the triangle taut. They position one leg of the triangle along the wall. This makes the other leg of the triangle perpendicular to the wall. The three members of the group holding the triangle slide it along the wall until the person standing at the designated spot declares that the leg of the triangle that is perpendicular to the wall lies on a line that passes directly over the designated spot. (From the perspective of the person standing at the designated spot, when the string triangle is in the correct position, then the vertex at the right angle of the triangle should be directly behind the “forward vertex” which is not along the wall.) Then the person standing at the right angle vertex should mark the position of this vertex on the wall.



We are now ready for the two main activities of this lesson: large-scale measurements of distance and height.

Activity 1. Each group will go outside and measure the height of a designated building with the help of a clipboard holder, using the idea described in Lesson 5 Homework Problem 1 (Dan's Problem). First choose a convenient position for the clipboard holder and mark a convenient reference height on the building. Measure the height of the reference mark above the ground using a measuring tape. (If necessary, stand on a chair to make the reference mark high enough and to measure its height.) Then use the clipboard holder at the designated spot to draw the appropriate figure. (As in Dan's Problem, the figure has four lines. One line runs from the eye to the bottom of the building, a second line runs from the eye to the reference height, a third line runs from the eye to the top of the building, and the fourth line is vertical.) Make appropriate measurements on the figure and calculate the height of the designated building using the appropriate formula. Report the result of the calculation to the instructor.

Activity 2. While still outside, each group will measure the distance from a designated point P on the ground to a designated wall of a nearby building with the help of a clipboard holder, using the idea described in Lesson 5 Homework Problem 2 (Hillary's Problem). First each group will employ the method described above (using the 3-by-4-by-5 string right triangle with washers at the vertices) to locate and mark a point Q on the wall that is closest to the designated point P. The group will then mark a second point R on the wall directly above Q at a convenient height. They will measure the distance between Q and R using a measuring tape. (If necessary, stand on a chair to make the point R high enough and to measure its distance above Q.) Then the group will use a clipboard holder to determine the distance from the designated point P to the designated wall. Place the clipboard holder at the designated point P and use it to draw an appropriate figure. (As in Hillary's Problem, the figure will have three lines. One line runs from the eye to the point Q, a second line runs from the eye to the point R, and the third line is vertical.) Make measurements on the figure and calculate the distance from the designated point P on the ground to the designated wall using the appropriate formula. Report the result of the calculation to the instructor.

Activity 3. The class will study and analyze the height and distance measurements calculated by the various groups in Activities 2 and 3 and try to account for variations in these results.

Homework Problem 1: Beth's Problem. Beth sees a sailboat in the distance which she knows has a mast that is 17 m tall. If Beth holds her arm straight in front of her so that her thumb is at eye level, and extends her thumb sideways, then the width of her thumb exactly covers the height of the mast. The width of Beth's thumb is 2.3 cm, and the distance from her eye to her thumb is 63.5 cm. Estimate the distance between Beth and the sailboat.

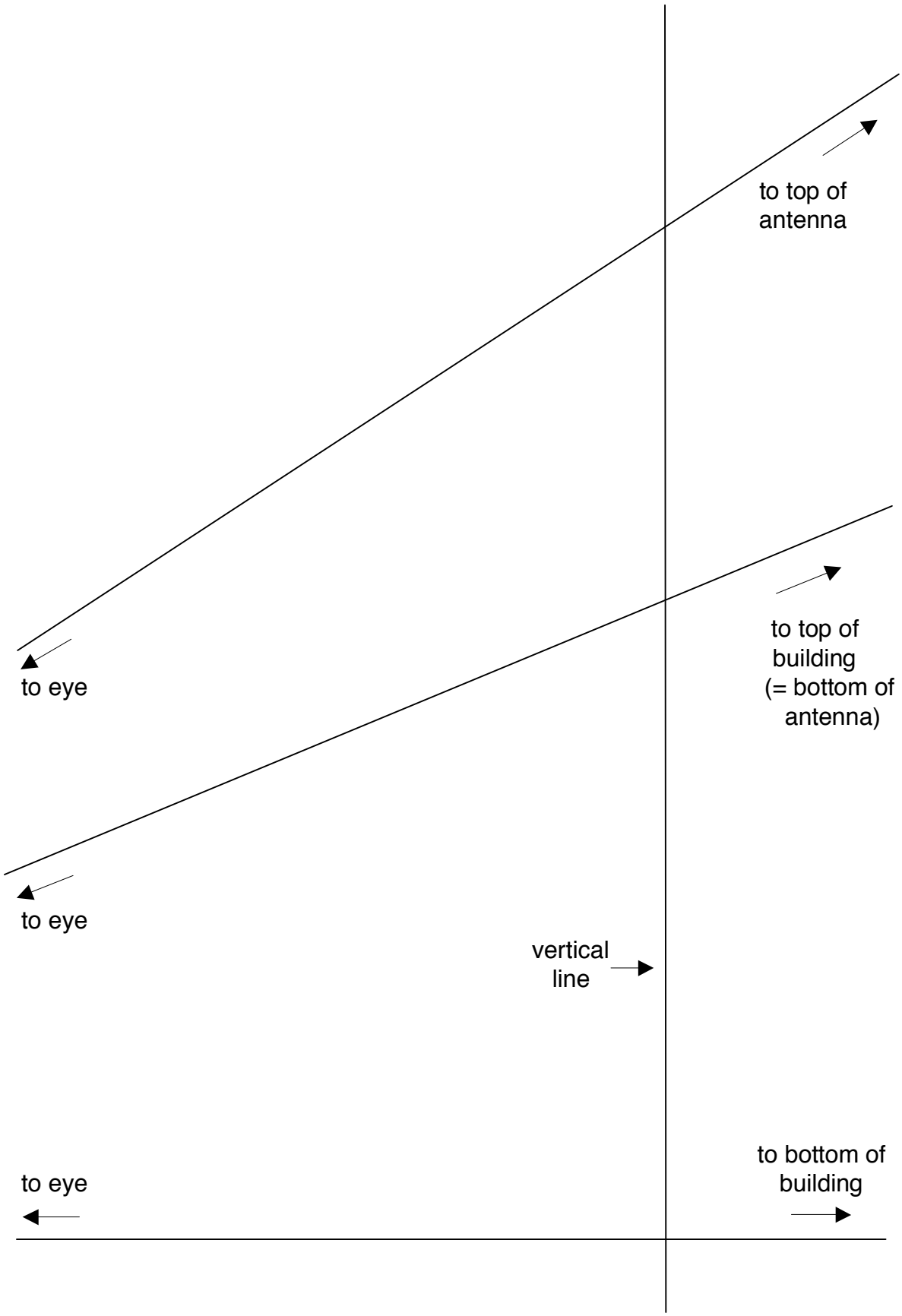
Homework Problem 2: Roxie's Problem. Roxie is standing in the middle of the Engineering Math Sciences Building lobby at UWM facing the building's front door, and she wants to measure both the her distance from the front door and the height of the lobby's ceiling. She has already measured the height of the front door and knows it is 2.1 m. She holds a meter stick vertically in front of her so that, from her perspective, the meter stick lines up with the front door, with the bottom of the door lining up with the 10 cm mark on the meter stick. The top of the door lines up with the 55 cm mark on the meter stick, and the line where the wall meets the ceiling lines up with the 74 cm mark on the meter stick. Also Roxie's eyes are 155 cm above the ground, and the distance from her eyes to the 10 cm mark on the meter stick is 64 cm. Roxie uses this information to calculate both her distance from the front door and the height of the ceiling. What answers do you think she should get?

Homework Problem 3: Hank's Problem. Hank knows that the radio antenna on top of one of the Sandburg Hall towers at UWM is 30 m high. He holds a clipboard with a piece of paper on it vertically in front of his eye and "aims" it at Sandburg Hall. He draws four lines on the paper:

- one line from his eye to the bottom of Sandburg Hall,
- one line from his eye to the top of Sandburg Hall (= the bottom of the radio antenna),
- one line from his eye to the top of the radio antenna, and
- one vertical line.

The figure he creates is on the next page. He measures some distances on the paper and calculates the height of Sandburg Hall (not including the antenna). Then he tapes a piece of paper to an edge of the page, draws more lines on the two taped pieces of paper, makes further measurements and calculates the distance from the point where he is standing to Sandburg Hall. Recreate Hank's procedure: tape a piece of paper to an edge of the next page, draw lines and making measurements on the figure on the two taped pages, and estimate the height of Sandburg Hall and the Hank's distance from Sandburg Hall.

Homework Problem 4. Instead of taping a piece of paper to the edge of the next page, use the *Generalized Distance Lemma* of Lesson 5 to determine Hank's distance from Sandburg Hall. Does this result agree with the corresponding result in Homework Problem 3?


to top of
antenna
to eye
to top of
building
(= bottom of
antenna)
to eye
vertical
line
to eye
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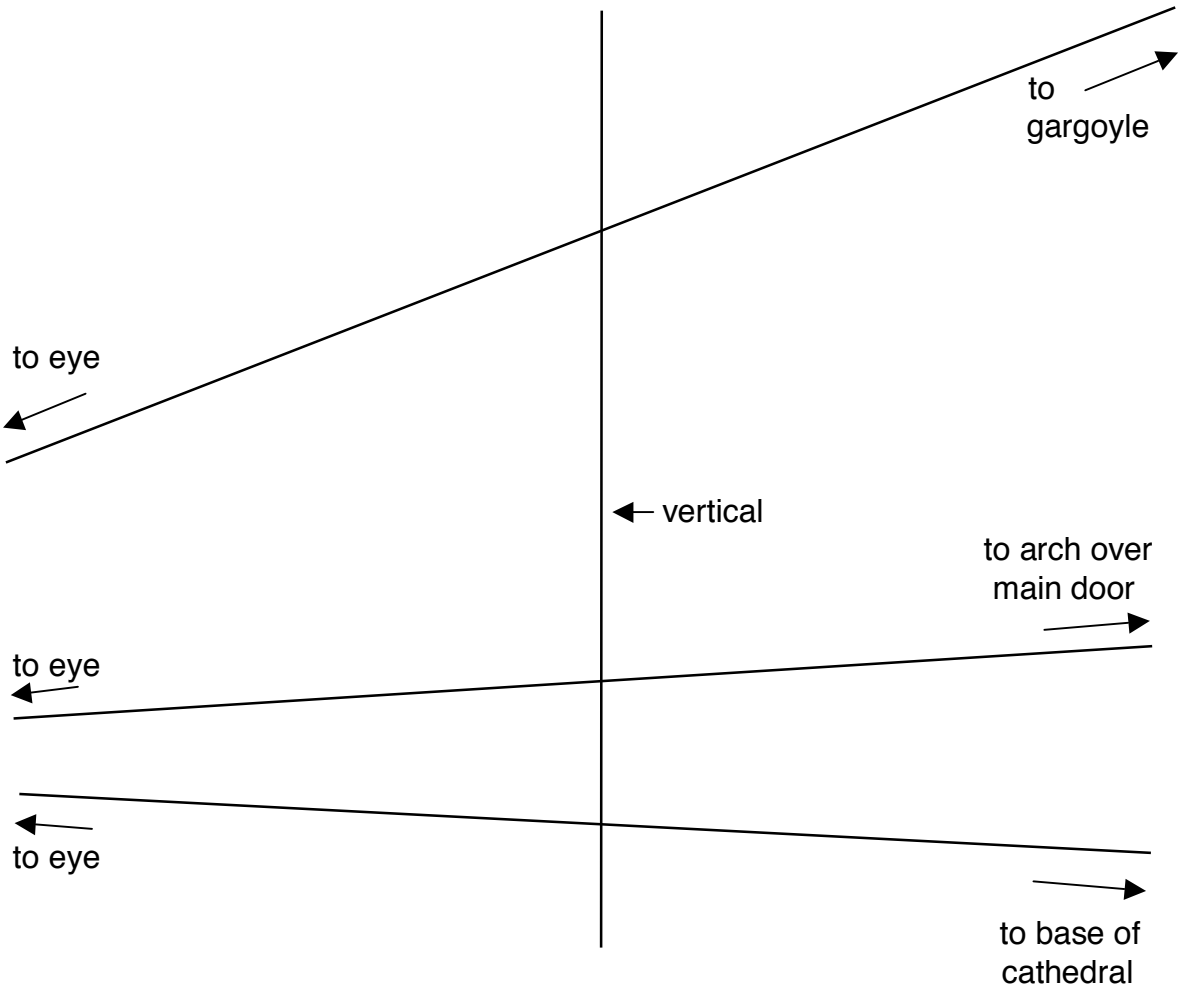
building

Homework Problem 5: Pandora's Problem. Pandora wants to measure the height of her favorite gothic cathedral. She learns from a guard that the arch over the main door of the cathedral is 4 m high and decides to use this measurement as a reference height. However, she finds that if she stands close enough to the cathedral to distinguish the base of the cathedral from the arch over the main door, then she can't easily see the top of the cathedral and can't easily draw a line from her eye to the top of the cathedral. However, if she stands far enough back from the cathedral to see the top, then the base and the arch over the main door appear so close that lines drawn from her eye to these two points essentially coincide. While pondering her problem, she notices a gargoyle perched part way up the front of the cathedral over the main door. Furthermore, she observes that she can stand close enough to the cathedral to distinguish the base and the arch and still easily see the gargoyle and draw a line from her eye to the gargoyle. Also she observes that she can stand far enough back from the cathedral to easily see the top and yet distinguish the base from the gargoyle and draw distinct lines from her eye to the base and to the gargoyle. These observations give her a good idea. She makes two figures. To create the first figure (on the next page), she stands closer to the cathedral and draws a vertical line and three lines from her eye to the base of the cathedral, to the arch over the main door, and to the gargoyle. To create the second figure (one the second page following this one), she stands farther from the cathedral and draws a vertical line and three lines from her eye to the base of the cathedral, to the gargoyle, and to the top of the cathedral. She then measures some distances in these two figures and calculates the height of the cathedral. What answer do you think she should get?


to
gargoyle
to eye
vertical
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main door
to eye
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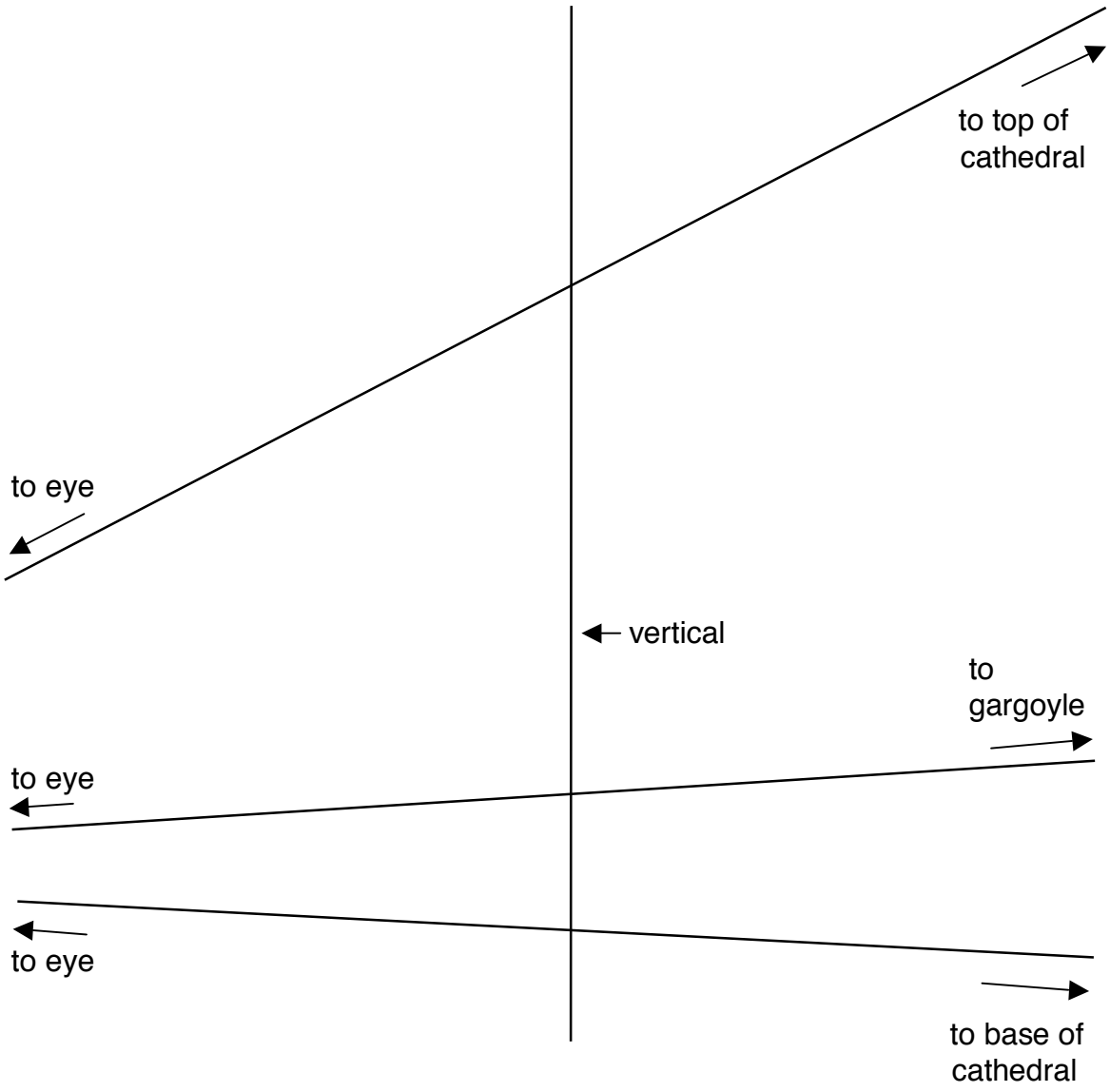
cathedral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cathedral
to eye
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gargoyle
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to eye
to base of
cathedral