Climate Dynamics (2021) 56:3305–3311
https://doi.org/10.1007/s00382-021-05646-7

Climate: a dynamical system with mismatched space and time domains

Peicai Yang[1] · Geli Wang[1] · Ziniu Xiao[2] · Anastasios A. Tsonis[3,4] · Guolin Feng[5] · Shida Liu[6] · Xiuji Zhou[7]

Received: 29 March 2020 / Accepted: 11 January 2021 / Published online: 14 February 2021

Abstract
Climate is a special spatiotemporal dynamical system. Its time scale can be extended indefinitely, but its space scale can never exceed that of the size of the system. We call this a "mismatching" in space and time domains. With the help of a simplified system of primitive equations, this exploratory paper shows that these scale characteristics may have a significant impact on the mathematical and physical structure of the system. The results show that the mismatching of space–time scales will lead to a decrease of the system's dimension, degenerating the system from an infinite dimensional to a finite one. Also they show that "mismatched" domains can lead to a greater consistency of the system's structure in space, as they form a system of uniform structures which are described as "patches". This may lead to an alternative way of representing climate and its variability as a pattern system defined by the collective behavior of interacting patches or subsystems.

Keywords Climate · Modeling · Space and time domains · Pattern dynamics · Climate subsystems

1 Introduction

Since the early atmospheric circulation climate models (see for example, Philips 1956), scientists have made great progress in developing and improving climate models. However, due to the complexity of the climate system, unprecedented difficulties (Held 2005; Latif 2011; Luan et al. 2016) still exist. Our intent here is not to discuss the state-of-the-art in climate modeling, but to present a theoretical analysis that may potentially lead to an alternative way of representing climate.

Atmospheric motion is a multi-scale space–time system. When a numerical model is used to simulate or to forecast, only specific space and time scales are considered (for example, a certain region of the planet and a certain length of time). In general, as a rough division, atmospheric processes can be divided into two different systems: weather and climate. Usually, atmospheric processes lasting less than two weeks (for example, mid-latitude systems) are called weather, and their corresponding spatial scales are clearly less than 10^4 km (Earth's circumference). Monthly and seasonal processes (such as those associated with temperature or precipitation anomalies) are called long-term weather processes or short-term climate processes. The atmospheric processes of years and greater (El Nino, decadal variability, and beyond) are collectively referred to as climate processes. The spatial scale corresponding to the latter two processes is still a very ambiguous problem, and it is also the issue to be discussed in this paper. Nevertheless, it is clear that in "weather" both the time scale and the space scale should be bounded. In "climate", however, the time scale can be extended indefinitely, while the spatial scale can only be confined to the size of the Earth's atmosphere itself. In this sense, the space and time domains may be regarded as "mismatched".

Peicai Yang
ypc@mail.iap.ac.cn

Geli Wang
wgl@mail.iap.ac.cn

1 LAGEO Institute of Atmospheric Physics, Chinese Academy of Sciences, Beijing, China

2 LASG Institute of Atmospheric Physics, Chinese Academy of Sciences, Beijing, China

3 University of Wisconsin-Milwaukee, Milwaukee, USA

4 Hydrologic Research Center, San Diego, USA

5 National Climate Center, Beijing, China

6 Department of Atmospheric and Oceanic Sciences, School of Physics, Peking University, Beijing, China

7 Chinese Academy of Meteorological Sciences, Beijing, China

About half a century ago, Fujita (1963) and Orlanski (1975) carefully classified the types of weather systems according to their time and space scales. They divided the most representative weather processes into three systems according to their spatial scales: macro, meso, and micro. Given the corresponding time scale range of these weather processes, it follows that the ratio of the spatial scale L to the time scale τ is proportional to a certain characteristic horizontal velocity, U (typically 10 m/*sec*), i.e.: $L/\tau \propto U$. Physically, this condition signifies that the energy and momentum carried by the weather system can maintain modes in a moderate time and space domain. Using the concept of "scale" to determine the basic dynamics of weather systems provides an important theoretical basis and a good starting point for the study of atmospheric motion.

This paper argues that the fact that the space domain is bounded while the time domain isn't will become an important scientific issue, and will have a far-reaching impact on our understanding of climate and its prediction. The main structure and contents of this paper are as follows: The following section uses a standard atmospheric dynamics model as a platform to discuss the relationship between the time and space scales, the resulting model structure. In the third section, we will present an example and we will discuss the implications of our findings. In the last section, we briefly summarize the main results of this paper.

2 The climate system and its space–time domain

Next, we will explore the consequences of the mismatch between the space and time domains by considering a simple model for atmospheric motion based on the conservation of momentum, mass, and energy laws.

$$\frac{\partial \vec{V}}{\partial t} + \vec{V} \bullet \nabla \vec{V} = -\frac{1}{\rho}\nabla p + \vec{g} - 2\vec{\omega} \times \vec{V} + \mu \nabla^2 \vec{V}$$

$$\frac{\partial \rho}{\partial t} + \vec{V} \bullet \nabla \rho = -\nabla \bullet \rho \vec{V} \tag{1}$$

$$p = \rho R_d T$$

$$\frac{\partial T}{\partial t} + \vec{V} \bullet \nabla T - \kappa \nabla^2 T = Q$$

The symbols used in the equation have the usual mathematical and physical meanings. Here, we can see that $\vec{V}$, p, ρ and T represent the velocity vector, pressure, density, and temperature of the atmosphere, while $\vec{g}$, $\vec{\omega}$, μ, κ, R_d and Q stand for the gravity acceleration, the rotational angular velocity of the earth, the atmospheric diffusion coefficient, the heat conduction coefficient, the ideal gas constant, and external heat source, respectively.

In order to facilitate the discussion on the scales issue, we have simplified the above equations by using the assumptions of hydrostatic equilibrium and incompressible flow and writing them in dimensionless form. These changes should not have an important impact on the analysis and conclusion of the problem:

$$\frac{\partial u^*}{\partial t^*} + S_1\left(\vec{V}^* \bullet \nabla^* u^* + \frac{\partial p^*}{\partial x^*} - R_o^{-1} v^* - R_e^{-1} \nabla^{*2} u^*\right) = 0 \tag{2.1}$$

$$\frac{\partial v^*}{\partial t^*} + S_1\left(\vec{V}^* \bullet \nabla^* v^* + \frac{\partial p^*}{\partial y^*} + R_o^{-1} u^* - R_e^{-1} \nabla^{*^2} v^*\right) = 0 \tag{2.2}$$

$$-\frac{\partial p^*}{\partial z^*} - g^* = 0 \tag{2.3}$$

$$\frac{\partial u^*}{\partial x^*} + \frac{\partial v^*}{\partial y^*} + \frac{\partial w^*}{\partial z^*} = 0 \tag{2.4}$$

$$p^* = \rho^* R_d^* T^* \tag{2.5}$$

$$\frac{\partial T^*}{\partial t^*} + S_1\left(\vec{V}^* \bullet \nabla^* T^* - R_p^{-1} \nabla^{*^2} T^*\right) = Q^* \tag{2.6}$$

Here $t^* = t/\tau$, $(x^*, y^*, z^*) = \left(x, y, \frac{L}{H}z\right)/L$, $\nabla^* = \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{H}{L}\frac{\partial}{\partial z}\right)L$, $(u^*, v^*, w^*) = \left(u, v, \frac{L}{H}w\right)\Big/U$, $\left(T^*, P^*, \rho^*\right) = \left(T/\Theta, P/\overline{\rho}U^2, \rho/\overline{\rho}\right)$, $g^* = \frac{gH}{U^2}$, $R_d^* = \frac{\Theta}{U^2}R_d$, $Q^* = Q\tau/\Theta$, while U, Θ, $\overline{\rho}$, and T, L, H stand for the characteristic quantities of horizontal wind speed, temperature, density and time scale, horizontal space scale and vertical space scale, respectively. The dimensionless parameters appearing in the equations, in addition to the well-known Rossby number $R_o = \frac{U}{fL}$ and Reynolds number $R_e = \frac{LU}{\mu}$ (including $R_p = \frac{LU}{\kappa}$), involve another parameter $S_1 = U\frac{\tau}{L}$ related to space–time scale and the characteristic quantity of atmospheric horizontal velocity U. Here, U can be expressed in terms of the macroscopic average of the horizontal wind speeds of the real atmosphere. Referring to the values used by Orlanski (1975) and Fujita (1963) in the scale classification of weather system, we assume a value of $U = 10$ m/*sec*.

Among the issues to be discussed in this paper, S_1 is a crucial parameter. It defines whether the system (2) is a weather process or a climate process. If, $S_1 \approx 1$ the system describes a weather process including atmospheric circulation. If $S_1 >> 1$, the system describes an atmospheric process with a finite spatial scale and a very large temporal scale. From Eqs. (2.1), (2.2), and (2.6), it can be seen that S_1 actually represents the ratio between the temporal variation and the spatial variation of the system. If $S_1 \approx 1$, it means that the contributions of these two parts to the system are balanced. We call such a system to be 'matched' in time and space domains. Conversely, if $S_1 >> 1$, the above balance is broken and the system is called 'mismatched'. As we will see below, the reduction in spatial variation caused by the mismatch would necessarily result in uniformity of the space state distribution of the system. Note that, between the above two cases, lies the interval where S_1 may be greater than one but not much greater than one, which represents the transitional states from weather to climate. We do not intend to discuss this issue here.

As we mentioned above, the set of equations we use here represent atmospheric motion. As such, this set does not represent the climate system per se, which is a coupled atmosphere–ocean-land system and involves slower oceanic and ice-sheet dynamics. This being an exploratory paper, we focus on the simpler atmospheric motion system for $S_1 >> 1$. Our hope is that our results will be followed by extension of this work to more complicated models.

For convenience in writing, we will omit the superscript "*" of all variables and will initially define the system (2) in a simple rectangular domain $D : \left(x_0 < x < x_1; y_0 < y < y_1; z_0 < z < z_1\right)$. To further simplify the system (2) we consider the conservation of energy Eq. (2.6). Because $S_1 >> 1$, it should hold that

$$-\left(\vec{V} \bullet \nabla T - R_{\mathrm{p}}^{-1}\nabla^2 T\right) = \frac{\left(\frac{\partial T}{\partial t} - Q\right)}{S_1} << 1 \tag{2.7}$$

If we further assume that $R_p^{-1} << 1$. This is equivalent to ignoring the heat conduction term in Eq. (2.6) (for this we only need to meet the condition $L >> k/U$). Then the above relationship should approximately result in the following equation.

$$\vec{V} \bullet \nabla T = 0 \tag{3}$$

The solution of Eq. (3) is an arbitrary function that is independent of space, and only depends on time (written as $T = T(t)$). This property of T is very important. It means that, within the spatial region D, the temperature T is uniformly distributed. This result provides us with an opportunity for further simplifying Eq. (2.6). In this case, the spatial average of Eq. (2.6) in the domain D gives:

$$\frac{\partial T}{\partial t} + S_1 \iiint \left(\vec{V} \bullet \nabla T\right) dxdydz = \overline{Q} \tag{4}$$

where $\overline{T} = \iiint T\mathrm{dxdydz} = \mathrm{T}$ and $\overline{Q} = \iiint Q\mathrm{dxdydz}$. Assuming that the vertical velocity takes a value of zero at the upper and lower boundaries of D i.e.$\mathrm{w}^{(z=z_0)} = \mathrm{w}^{(z=z_1)} = 0$ (note that $\mathrm{w}^{(z=z_B)} = \mathrm{w}\left(\mathrm{x,y,z_B}\right)$), we obtain the integrals for the second term in Eq. (4):

$$\iiint \left(\frac{\partial \mathrm{uT}}{\partial x} + \frac{\partial \mathrm{vT}}{\partial y} + \frac{\partial \mathrm{wT}}{\partial \mathrm{z}}\right)\mathrm{dxdydz} = \overline{uT}^{(\mathrm{x=x_1})} - \overline{uT}^{(\mathrm{x=x_0})} + \overline{vT}^{(y=y_1)} - \overline{vT}^{(y=y_0)} \tag{5}$$

Here $\overline{uT}^{(\mathrm{x=x_1})} = \iint u\left(x_1, y, z, t\right)T\left(x_1, y, z, t\right)dydz$ (the other three terms are defined similarly). These four terms in the right-hand of Eq. (5) represent the averages of temperature flux on boundaries (x_1, y, z), (x_0, y, z) (x, y_1, z) and (x, y_0, z) of D, respectively. They are all function of time and can be given by boundary conditions. It is not difficult to see that Eq. (5) is the result given by the famous Gauss law. This result indicates that the average value of temperature transport in domain D is equal to the difference of the average temperature fluxes on both sides of the boundary of D. Using this property, we can transfer some physical quantities in the domain D to the boundary and discuss them with other corresponding physical quantities.

After these results are substituted into Eq. (4), the nonlinear ODE describing the temperature T in D can be established as follows:

$$\frac{\mathrm{d}T}{\mathrm{d}t} + S_1\left(\overline{uT}^{(\mathrm{x=x_1})} - \overline{uT}^{(\mathrm{x=x_0})} + \overline{vT}^{(y=y_1)} - \overline{vT}^{(y=y_0)}\right) = \overline{Q} \tag{6}$$

Noted that the condition $\left(\vec{V} \bullet \nabla T\right) \ll 1$ (or ≈ 0) does not mean that $S_1\left(\vec{V} \bullet \nabla T\right) \ll 1$ (or ≈ 0). This is because both S_1 and $1\Big/\left(\vec{V} \bullet \nabla T\right)$ are variables of the same order (see Eq. (2.7)). Following the above steps, we can also construct a differential equation describing the mean horizontal wind field $\left(\overline{u}, \overline{v}\right)$ by means of (2.1) and (2.2). It is not difficult to see that, assuming the quasi-geostrophic approximation and $R_o^{-1} << 1$, the Eqs. (2.1) and (2.2) give the following relationships

$$\left|\vec{V} \bullet \nabla u\right| = \frac{\left|\frac{\partial u}{\partial t}\right|}{S_1} << 1$$

$$\left|\vec{V} \bullet \nabla v\right| = \frac{\left|\frac{\partial v}{\partial t}\right|}{S_1} << 1$$

Furthermore, we can obtain the following equations

$$\begin{aligned} \mathrm{u}\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} = 0 \\ \mathrm{u}\frac{\partial \mathrm{v}}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z} = 0 \end{aligned} \tag{7}$$

The above equations indicate that inside the domain D the horizontal wind field is also spatially uniform and only a function of time. Neglecting the friction term and implementing spatial averaging for Eqs. (2.1) and (2.2), we obtain that:

$$\begin{aligned}
&\frac{\partial \mathrm{u}}{\partial t}+S_1\left(\iiint \vec{V}\bullet\nabla u dxdydz+\iiint\left(\frac{\partial p}{\partial x}-R_o^{-1}v\right)\mathrm{dxdydz}\right)=0\\
&\frac{\partial \mathrm{v}}{\partial t}+S_1\left(\iiint \vec{V}\bullet\nabla v dxdydz+\iiint\left(\frac{\partial p}{\partial y}+R_o^{-1}u\right)\mathrm{dxdydz}\right)=0
\end{aligned}\tag{8}$$

Due to the uniformity of (u,v) in domain D, we may derive the ODEs that the mean wind speeds $\overline{\mathrm{u}}$ and $\overline{\mathrm{v}}$ satisfy:

$$\begin{aligned}
&\frac{\mathrm{du}}{dt}+\mathrm{S}_1\alpha_1=0\\
&\frac{dv}{dt}+\mathrm{S}_1\alpha_2=0
\end{aligned}\tag{9}$$

where

$$\begin{aligned}
\alpha_1&=\iiint\left(\vec{V}\bullet\nabla u+\frac{\partial \mathrm{p}}{\partial x}-\mathrm{R_o^{-1}v}\right)dxdydz=\overline{u^2}^{(\mathrm{x=x_1})}-\overline{u^2}^{(\mathrm{x=x_0})}+\overline{uv}^{(\mathrm{y=y_1})}-\overline{uv}^{(\mathrm{y=y_0})}+\overline{\mathrm{p}}^{(\mathrm{x=x_1})}-\overline{\mathrm{p}}^{(\mathrm{x=x_0})}-\mathrm{R_o^{-1}v}\\
\alpha_2&=\iiint\left(\vec{V}\bullet\nabla \mathrm{v}+\frac{\partial \mathrm{p}}{\partial y}+\mathrm{R_o^{-1}u}\right)dxdydz=\overline{v^2}^{(\mathrm{y=y_1})}-\overline{v^2}^{(\mathrm{y=y_0})}+\overline{uv}^{(\mathrm{x=x_1})}-\overline{uv}^{(\mathrm{x=x_0})}+\overline{\mathrm{p}}^{(\mathrm{y=y_1})}-\overline{\mathrm{p}}^{(\mathrm{y=y_0})}+\mathrm{R_o^{-1}u}
\end{aligned}\tag{10}$$

Furthermore, multiplying both sides of the Eq. (2.1) and (2.2) by $2u$ and $2v$, respectively, and then adding both of them, we can obtain the kinetic energy equations of the system

$$\frac{\partial E^2}{\partial t}+\mathrm{S}_1\left(\vec{V}\bullet\nabla R\right)=0\tag{11}$$

where $E^2=\mathrm{u}^2+v^2$, $R=E^2+2p+2gz$. Because $S_1>>1$, then we should have $\vec{V}\bullet\nabla R=0$,which gives

$$R=E^2+2p+2\mathrm{gz}=\mathrm{c}_1(t)$$

where $c_1(t)$ is an arbitrary time function. Then, calculation of its spatial partial derivative gives

$$\begin{aligned}
&2\frac{\partial p}{\partial x}=-\frac{\partial \mathrm{E}^2}{\partial x}=-2\left(u\frac{\partial u}{\partial x}+v\frac{\partial v}{\partial x}\right)\\
&2\frac{\partial \mathrm{p}}{\partial y}=-\frac{\partial \mathrm{E}^2}{\partial y}=-2\left(u\frac{\partial u}{\partial y}+v\frac{\partial v}{\partial y}\right)
\end{aligned}\tag{12}$$

The above equations indicate that the horizontal pressure gradient field of the system is also uniform in space. Because of Eq. (12), Eq. (10) becomes

$$\begin{aligned}
\alpha_1&=\frac{1}{2}\left(\overline{u^2}^{(\mathrm{x=x_1})}-\overline{u^2}^{(\mathrm{x=x_0})}-\overline{v^2}^{(\mathrm{x=x_1})}+\overline{v^2}^{(\mathrm{x}=x_0)}\right)+\overline{uv}^{(\mathrm{y=y_1})}-\overline{uv}^{(\mathrm{y=y_0})}-\mathrm{R_o^{-1}v}\\
\alpha_2&=\frac{1}{2}\left(\overline{v^2}^{(\mathrm{y=y_1})}-\overline{v^2}^{(\mathrm{y=y_0})}-\overline{u^2}^{(\mathrm{y=y_1})}+\overline{u^2}^{(\mathrm{y=y_0})}\right)+\overline{uv}^{(\mathrm{x=x_1})}-\overline{uv}^{(\mathrm{x=x_0})}+\mathrm{R_o^{-1}u}
\end{aligned}\tag{13}$$

So far, we defined a domain D and derived a set of ordinary differential equations composed of Eqs. (6), (9), and (13), which describe, in terms of the temperature, the horizontal wind, and the air pressure, the climate in that domain when the space and time scales are "mismatched". The above analysis reveals the important influence of the mismatch of the space–time scales on the mathematical physical structure of the atmospheric system, more specifically, that the increase in mismatch necessarily leads to the uniformity (in terms of mean temperature, mean wind, and mean pressure) of the system's structure in space. Next, some more interesting results from this are presented.

3 Pattern climate

With the help of the mathematical and physical framework presented above, we propose a new climate system which we call "pattern climate".

We begin by dividing the global region into k sub-regions according to the difference in some driving force, and we call each sub-region a 'patch' (denoted as$D_i|_{\mathrm{i=1,\ldots,k}}$). For each patch, a corresponding climate subsystem is established according to Eq. (2). Based on the results presented in Sect. 2, we assume that the scale parameter $S_1^{(\mathrm{i})}$ and the driving force Q_i of each subsystem are mismatched and only depend on time not on space. We also assume that the state variables of the system remain continuous on the boundary Γ between adjacent patches. This allows for exchange of energy and mass. While what exactly the nature of the driving force is may be open to further research, several options may be available. For example, the spatial distribution of heat or winds, or even known large scale features, such as El Nino, NAO, etc., could be considered.

The above formulation may be reminiscent of the geographical classifications of climatology, but there is more to our results than climate classification. Eqations (6), (9), and (13) represent a set of nonlinear ODEs that describe the dynamics inside a patch. In a sense, these equations represent a much lower dimensional dynamical subsystem. And a group of patches can form a 'climate pattern', where communication between adjacent patches is possible, thereby defining global climate variability. This is similar to recent developments in network theory and its applications to climate (for example, Tsonis et al. 2010; Steinhaeuser and Tsonis 2013; Fountalis et al. 2013). They have shown that climate fields (both observed and simulated) are composed of a number of interacting communities or subsystems, with each community obeying distinct dynamics or rules and therefore subject to different driving forces. The overall network of communities (which can be also thought as "patches") and their interactions explain fundamental features of the climate system. We note here that other approaches to produce reduced climate models have been proposed in the past, but not dealing with the mismatch of the time and space domains (see for example, Majda et al. 2009, where stochastic reduced climate models are suggested for the study of atmospheric low-frequency variability). In addition, it should be pointed out that the case of k = 1 is an interesting special one, corresponding to a climate system which contains one global super patch. Scientists have done a lot of research for it. Using the principle of balance of atmospheric radiation energy budget, they established some climate systems called "box model" and discussed the nonlinear dynamic mechanism of long-term climate process (for example, North and Coakley 1979; Saltzman 2001; Duan and Zhou 2014).

As an example, we construct a simple climate pattern system consisting of three patches D_1, D_2 and D_3, each of which is adjacent to the other two (see Fig. 1). We use $L_i(i = 1, 2, 3)$ represent the spatial scale of, the corresponding patch, while $l_i(i = 1, 2, 3)$ to indicate the length of the a, djacent sides of the two patches.

Here, we only give the equations on the temperature T. Following the steps given in the Sect. 2, the construction of this climate model should satisfy the following average equations:

$$\begin{cases} \frac{\partial T_1}{\partial t} + S_1^{(1)} \iiint\limits_{D_1} \left(\vec{V}_1 \bullet \nabla T_1\right) dxdydz = \overline{Q}_1 \\ \frac{\partial T_2}{\partial t} + S_1^{(2)} \iiint\limits_{D_1} \left(\vec{V}_2 \bullet \nabla T_2\right) dxdydz = \overline{Q}_2 \\ \frac{\partial T_3}{\partial t} + S_1^{(3)} \iiint\limits_{D_1} \left(\vec{V}_3 \bullet \nabla T_3\right) dxdydz = \overline{Q}_3 \end{cases} \tag{14}$$

where $S_1^{(i)} = U\tau / L_i | i = 1, 2, 3$ Carrying out the integral (see Eq. 6) and considering the continuous boundary condition, Eq. (14) becomes

$$\begin{cases} \frac{\mathrm{d}T_1}{\mathrm{d}t} + \mathrm{a}_1 S_1^{(1)} \left(u_2 T_2 - u_3 T_3\right) = \overline{Q}_1 \\ \frac{\mathrm{d}T_2}{\mathrm{d}t} + \mathrm{a}_2 S_1^{(2)} \left(u_3 T_3 - u_1 T_1\right) = \overline{Q}_2 \\ \frac{\mathrm{d}T_3}{\mathrm{d}t} + \mathrm{a}_3 S_1^{(3)} \left(u_1 T_1 - u_2 T_2\right) = \overline{Q}_3 \end{cases} \left(a_\mathrm{i} = \frac{l_\mathrm{i}}{L_i} |_{\mathrm{i}=1,2,3} \right) \tag{15}$$

Here, a_i can be regarded as the internal energy exchange coefficient determined by the size of the of the patch interface. Adding the above three equations, we can get

$$\frac{\mathrm{d}\left(\sum_{i=1,2,3} c_i T_i\right)}{\mathrm{d}t} = \sum_{i=1,2,3} c_i \overline{Q}_i \tag{16}$$

where $\mathrm{c}_1 = \mathrm{a}_2 a_3 S_1^{(2)} S_1^{(3)}$, $\mathrm{c}_2 = \mathrm{a}_3 a_1 S_1^{(3)} S_1^{(1)}$ and $\mathrm{c}_3 = \mathrm{a}_1 a_2 S_1^{(1)} S_1^{(2)}$. The above equation indicates that the tendency of the total internal energy of the three patches depends only on the total external source, where c_i are their respective weights.

We can extend Eqs. (15) and (16) to a global pattern consisting of M patches. This system is equivalent to place all the patches on a closed sphere and completely covering it. Here any one of the patches D_i ($i = 1, 2, \ldots, M$) and its adjacent k patches can form a 'patch group' (the schematic view can be seen in Fig. 2).

In this case, the global climate system consisting of all patches can be expressed as follows

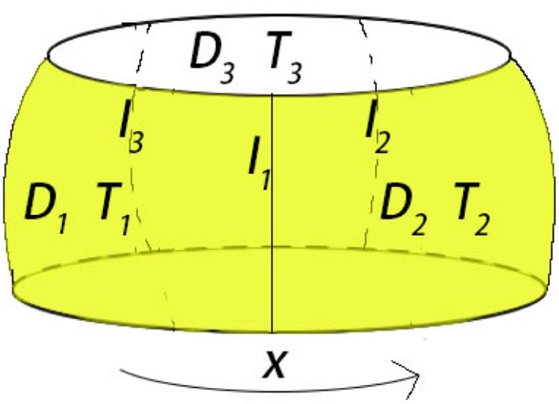

Fig. 1 A schematic diagram of pattern system consisting of three patches

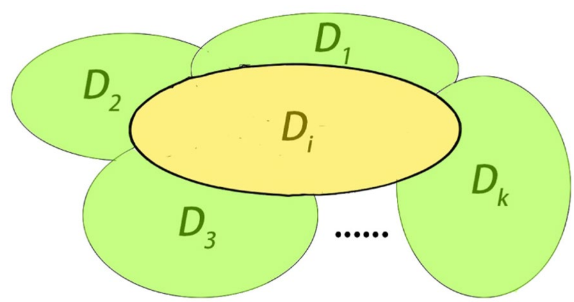

Fig. 2 Schematic diagram of the patch group which is consisting of D_i and its adjacent patches D_j ($j = 1, 2\ldots, k$); k is the number the adjacent patches

$$\frac{\mathrm{d}T_i}{\mathrm{d}t} + \beta_i \sum_{j=1}^{k} \mathrm{V}_{\mathrm{i,j}}^{(\mathrm{n})} T_j = Q_i \quad (\mathrm{i} = 1, ..., \mathrm{M}) \tag{17}$$

where $\mathrm{V}_{\mathrm{i,j}}^{(\mathrm{n})}$ $(i=1, 2,\ldots, M; j=1,2,\ldots, k)$ can be given by Eqs. (9) and (13). They represent the normal wind component of patch D_j at its junction with the patch D_i, which is defined as positive when it blows toward D_i. $\beta_i = \frac{\mathrm{l_i}}{\mathrm{L_i}} S_1^{(i)}$ $\beta_\mathrm{i} = \frac{\mathrm{l_i}}{\mathrm{L_i}} \mathrm{S}_1^{(\mathrm{i})}$ represents the flux exchange coefficient between D_i and D_j, and M is the total number of the patches or patch groups.

In essence, Eq. (17) can be seen as a climate subsystem. Each equation is determined by its own spatial scale L and driving force Q. The set of subsystems constitutes a pattern system. Therefore, we can think that the non-uniformity of the spatial distribution of the driving force creates the climatic pattern.

Similar to Eq. (16), we also have that $\sum_{i=1}^{\mathrm{M}} \frac{d\mathrm{c_i}T_i}{dt} = \sum_{i=1}^{\mathrm{M}} \mathrm{c_i} Q_i$ or $\frac{d\tilde{T}}{dt} = \tilde{Q}$, where $\tilde{T}$ and $\tilde{Q}$ represent the total internal energy and total external source of the system, respectively, which indicate that the total internal energy of the system is independent of the energy exchange between the patches. Note that analogous to Eq. (17), equations corresponding to the mean horizontal wind and mean air pressure can be derived from Eqs. (9) and (12).

In addition, the irregular spatial structure can also be built into a regular one. For example, if we cover it with a regular net, and as long as the condition $S_1 >> 1$ applies, then, following the Eq. (17), we can establish a climate model consisting of the regular patch groups as follows (also see Fig. 3).

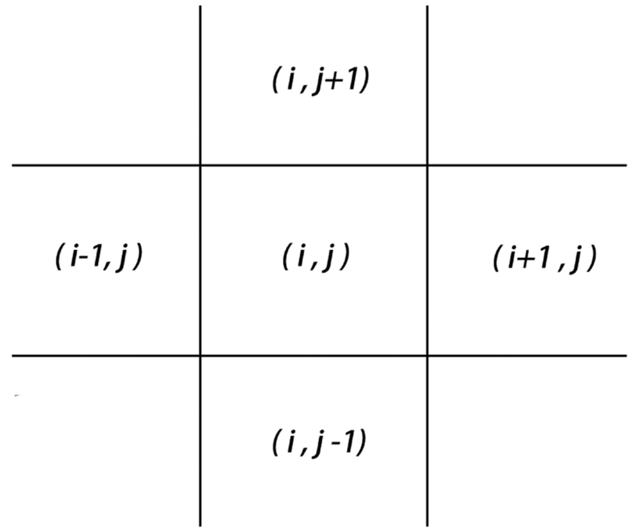

Fig. 3 Schematic diagram of the regular mesh, where (i, j) stands for the mesh coordinates

and corresponding to the weather process, while $S_1 >> 1$ indicates that the two domains are "mismatched" and corresponding to the climate process. The atmospheric system between the two above cases is considered as a long-term weather process or short-term climate process. The increase of S_1 means that the influence of nonlinear spatial transport in the system is weakened, as the system tends more and more to uniformity. When S_1 is large enough, the spatial structure of the system state will homogenize and form a "patch". Each patch is open. It can exchange energy

$$\frac{dT_{i,j}}{dt} + S_1\left[\left(u_{i+1,j}T_{i+1,J} - u_{i-1,j}T_{i-1,j}\right) + \left(v_{i,j+1}T_{i,j+1} - v_{i,j-1}T_{i,j-1}\right)\right] = \overline{Q}_{i,j}\,(i = 1, \ldots, m; j = 1, \ldots, m) \tag{18}$$

where $\overline{Q}_{i,j}$ stands for the average of the external source in the mesh $(i; j)$.

and mass with its adjacent patches and therefore make up of a patch group. All the patches form a "pattern".

4 Conclusion and discussion

As a brief summary of this paper, we give the following conclusions: Compared with weather system, climate is a very special space–time system. Its time scale can be arbitrarily long, but its spatial scale is always limited. This characteristic of the space and time domains leads to a variety of climate states. In this paper, a dimensionless parameter $S_1 = U\tau/L$ is used to represent the structures of the space and time scales of the atmospheric system. $S_1 \approx 1$ indicates that the space–time domains are "matched"

As we mentioned in the beginning of this paper, this work is an exploratory work into an idea that may lead to an alternative way to view and model climate variability. It is by far not complete, and more research is needed to further support our results. For example, observed and climate simulated fields could be considered, derive the patches from the communities of the corresponding networks and test our ideas. Work in this area is underway and we hope to report results in the near future.

Acknowledgements This research was partially supported by the National Key R&D Program of China (2017YFC1501804) and the National Natural Science Foundation of China (42075054, 91737102 and 41575058). Useful discussions with Dr. Tianjun Zhou and Dr. Cheng-Zhi Zou are gratefully acknowledged. The authors are grateful to the three anonymous referees and Dr. Tamas Bodai for their useful comments that helped to improve the quality of the manuscript.

References

Duan MK, Zhou XJ (2014) Stochastic dynamic simulation of the 100-kyr cycles in climate system. Sci China Earth Sci 58(3):420–428

Fountalis I, Bracco A, Dovrolis C (2013) Spatio-temporal network analysis for studying climate patterns. Clim Dyn. https://doi.org/10.1007/s00382-013-1729-5

Fujita T (1963) Analytical mesometeorology: a review, severe local storms. Meteoral Monogr 27:77–128. https://doi.org/10.1007/978-1-940033-56-3-5

Held IM (2005) The gap between simulation and understanding in climate modeling. Bull Am Meteoral Soc. https://doi.org/10.1175/BAMS-86-11-1609

Latif M (2011) Uncertainty in climate change projections. J Geochem Explor 110:1–7

Luan YH, Yu YQ, Zheng WP (2016) Review of development and application of high resolution global climate system model. Adv Earth Sci 31(3):258–268. https://doi.org/10.11867/j.issn.1001-8166.2016.03.0258

Majda AJ, Christian Franzke C, Crommelin D (2009) Normal forms for reduced stochastic climate models. Proc Natl Acad Sci 106(10):3649–3653. https://doi.org/10.1073/pnas.0900173106

North GR, Coakley JA (1979) Differences between seasonal and mean annual energy balance model calculations of climate and climate sensitivity. J Atmos Sci. https://doi.org/10.1175/1520-0469(1979)0362.0.CO;2

Orlanski I (1975) A rational subdivision of scales for atmospheric processes. Bull Am Meteorol Soc. https://doi.org/10.1155/2013/525383

Philips NA (1956) The general circulation of the atmosphere: a numerical experiment. Q J R Meteorol 352(82):123–164. https://doi.org/10.1002/qj.49708235202

Saltzman B (2001) Dynamical paleoclimatology: generalized theory of global climate change. Academic Press, 354

Steinhaeuser K, Tsonis AA (2013) A climate model intercomparison at the dynamics level. Clim Dyn. https://doi.org/10.1007/s00382-013-1761-5

Tsonis AA, Wang G, Swanson K, Rodrigues FA, da Fontura Costa L (2010) Community structure and dynamics in climate networks. Clim Dyn 37:933–940. https://doi.org/10.1007/s00382-010-0874-3