



Structure and Properties of the Attractor of a Marine Dynamical System

G. N. TRIANTAFYLLOU*

Department of Geosciences, University of Wisconsin-Milwaukee

P.O. Box 413, Milwaukee, WI 53201, U.S.A.

g20csd4.csd.uwm.edu

J. B. ELSNER

Department of Meteorology, Florida State University

Tallahassee, FL 32306, U.S.A.

A. LASCARATOS

Department of Applied Physics, University of Athens

33 Ippokratous St., Athens 106.80, Greece

C. KOUTITAS

Department of Civil Engineering, Aristotle University of Thessaloniki

Thessaloniki 540.06, Greece

A. A. TSONIS

Department of Geosciences, University of Wisconsin-Milwaukee

Milwaukee, WI 53201, U.S.A.

(Received and accepted April 1994)

Abstract—We investigate the properties of a marine dynamical system by means of time series of the sea-level height at four locations in the Saronikos Gulf in the Aegean Sea, Greece. In order to characterize the dynamics, we estimate the dimension of the underlying system attractor, and we compute its Lyapunov exponents. Dimension estimates indicate that the dynamics can be explained by a low-dimensional deterministic dynamical system. Lyapunov exponent estimates further substantiate the above conclusion, while at the same time, indicate that the dynamical system is a rather nonuniform chaotic one.

Keywords—Marine dynamical systems, Chaos, Nonlinear systems, Fractal dimension.

1. INTRODUCTION

Lately, ideas from the theory of nonlinear dynamical systems and chaos have been applied to many problems from many different disciplines, including oceanographic and atmospheric sciences. The main goal is the search for low-dimensional chaos and the extraction of the properties of the underlying attractors, if any. The procedure often involves an observable (time series) and a reconstruction of the attractor. The reconstruction is achieved by taking a scalar time series $\chi(t_i)$ and its successive time shifts (delays) as coordinates of a vector time series given by:

$$\mathbf{X}(t_i) = \{\chi(t_i), \chi(t_i + \tau), \dots, \chi(t_i + (n-1)\tau)\}, \quad (1)$$

*Author to whom all correspondence should be addressed.

where n is the dimension of the vector $\mathbf{X}(t_i)$ (often referred to as the *embedding dimension*) and τ is an appropriate delay [1–3].

For an n -dimensional phase space, a “cloud” or a set of points will be observed. From this set, the various dimensions and exponents that characterize the underlying attractor can be calculated. The concepts behind such procedures have been developed and presented extensively over the past five years in numerous papers and books (see for example [4] and references herein). Therefore, we will present the major ideas behind these procedures, and we will avoid lengthy discussions.

The most popular approach is to calculate the *correlation dimension*. According to this approach [5,6], given the cloud of points one finds the number of pairs $N(r, n)$ with distances less than a distance r . In this case, if for significantly small r , we find that:

$$N(r, n) \propto r^{d_2}, \quad (2)$$

then the scaling exponent d_2 is the correlation dimension of the attractor for that n . Since the dimension of the underlying attractor is not known, we test equation 2 for increasing values of n and check for a saturation value D_2 , which will be an estimation of the correlation dimension of the attractor. For more details on the aforementioned procedures and the applications related to weather, climate and other areas, see the review articles by Tsonis [4] and Tsonis and Elsner [7,8].

It should be mentioned at this point that there are no certain rules for choosing τ . When we reconstruct the attractor by producing a cloud of points at a given embedding dimension, those points should be independent to each other. Otherwise, $\chi(t + \tau) \approx \chi(t)$ and the points tend to fall on the diagonal. As a result, the estimation of the correlation dimension may be biased (typically underestimated). Therefore, τ must be chosen so as to result in points that are not correlated to previously generated points. Thus, a first choice of τ should be in terms of the decorrelation time of the time series under investigation. The question now arises: How do we define the decorrelation time? A straightforward procedure is to consider the decorrelation time equal to the lag at which the autocorrelation function for the first time attains the value of zero. Other approaches consider the lag at which the autocorrelation function attains a certain value like $1/e$, 0.5 [9], or 0.1 [10]. Another suggestion for the choice is to take τ equal to T/n where T is the dominant periodicity (as revealed by Fourier analysis) and n is the embedding dimension. In this way τ gives some measure of statistical independence of the data average over an orbit, and it is an appropriate approach if the autocorrelation function is periodic. As it was pointed out, however, by Frazer and Swinney [11] the autocorrelation function measures the linear dependence among successive points and may not be appropriate when we are dealing with nonlinear dynamics. They argue that what should be used as τ is the local minimum of the mutual information that measures the general dependence among successive points. Evidently, no one of the aforementioned rules has emerged as the undisputed rule for choosing τ , but the mutual information approach appears to have the edge. Nevertheless, a very reassuring practice is experimenting with various τ 's (while repeating the aforementioned constraints) in order to address possible effects of the choice of τ .

Another set of exponents that can characterize the properties of an attractor of a dynamical system are the Lyapunov exponents. The Lyapunov exponents are related to the average rates of convergence and/or divergence of nearby trajectories in phase space and, therefore, they measure how predictable or unpredictable the system is.

Consequently, the Lyapunov exponents are defined as follows. We start with a set of initial conditions in an attractor (embedded in an n -dimensional Euclidean space) that are confined within an n -dimensional sphere. Subsequently, we begin to monitor the long time evolution of this sphere. We order the principal axes of this sphere from the most rapidly to the least rapidly growing, and we compute the mean growth rate λ_i of any given principal axis p_i . We may define

these growth rates as follows:

$$\lambda_i = \lim_{T \rightarrow \infty} \frac{1}{T} \int_0^T dt \frac{d}{dt} \ln \left[\frac{p_i(t)}{p_i(0)} \right] = \lim_{T \rightarrow \infty} \frac{1}{T} \ln \left[\frac{p_i(T)}{p_i(0)} \right].$$

Here, $p_i(0)$ is the radius of the principal axis p_i at $t = 0$ (i.e., in the initial hypersphere) and $p_i(t)$ its radius after a long time T . The set of λ_i 's is referred to as the *Lyapunov exponent spectrum*. Apparently, there are as many Lyapunov exponents as the dimension of the phase space.

When at least one Lyapunov exponent is positive, then the system at hand is chaotic and the initial sphere will evolve to some complex ellipsoid structure reflecting the exponential divergence of nearby initial conditions along at least one direction on the attractor. This "sensitivity" to the initial conditions results in an inability to predict the evolution of the trajectory beyond an interval of time of the order of the inverse of the divergence rate. When no positive Lyapunov exponent exists, then no exponential divergence exists, and thus, the long-term predictability of the system at hand is guaranteed. In a way, we may say that the exponents measure the rate at which the system destroys information. Positive exponents give an idea of how fast information contained in a set of initial conditions initially very close to each other is lost due to the action of the stretching and folding of the chaotic attractor and negative exponents give an idea about the average rate at which information contained in transients is lost. Thus, in a sense, negative and positive exponents define dominant time scales in the evolution of a dynamical system (see for example [12]).

With the notable exception of the work of Osborne *et al.*, [13], Provenzale *et al.*, [14] who investigated the chaotic behavior in large and mesoscale motions in the Pacific Ocean, not much work has been done in applying ideas from the theory of dynamical systems in oceanography. In other fields such as the closely related areas of atmospheric sciences ([7,10,12,15–19], etc.) there has been a lot of activity in applying these ideas and low-dimensional attractors have been suggested to underline weather observables like geopotential, wind, rainfall, etc. One of the critical and much debated issues in these and other studies that used the above techniques to reconstruct the attractor underlying an observable is the length of the observable record. In other words, how many points are needed to confidently estimate the various dimensions and Lyapunov exponents. Early theoretical studies suggest that the number of points should be of the order of 10^d or 42^d where d is the dimension of the underlying attractor (which one tries to calculate) [20,21]. Lately, however, it is being realized that the required number of points might be significantly less and approximately of the order of $10^{2+0.4d}$ for a confidence level of 95% [22]. Because of the above "theoretical" uncertainties a certain procedure must be followed in order to ensure the existence of finite dimensions and Lyapunov exponents. These procedures will become clear as we proceed with the presentation of our results.

2. RESULTS

Here we apply the above methods to data which represent observables from a marine dynamical system. These observables show the variation of the sea-level (as measured from the bottom of the sea) as a function of time at four stations (Souvala, Ag.Marina, Vouliagmeni and Salamina) in the Saronikos Gulf in the Aegean Sea (see Figure 1). The measurements were made from July 1986 to September 1987 by the University of Athens using Aanderaa WLR-5 bottom pressure gauges. Each record is longer than a year with the exception of Ag.Marina. The temporal resolution of the records is 10 minutes. Part of the dynamics of the sea level are the waves generated by the wind. This time scale is relatively small and thus the sampling time of 10 minutes is not considered too short. Specifically, the total number of data points for each station is 61,583 (Souvala), 49,400 (Ag.Marina), 60,046 (Vouliagmeni) and 62,184 (Salamina). Thus, a concatenated time series (including information from all four stations) will consist of 233,213 data points. We should mention at this point that:

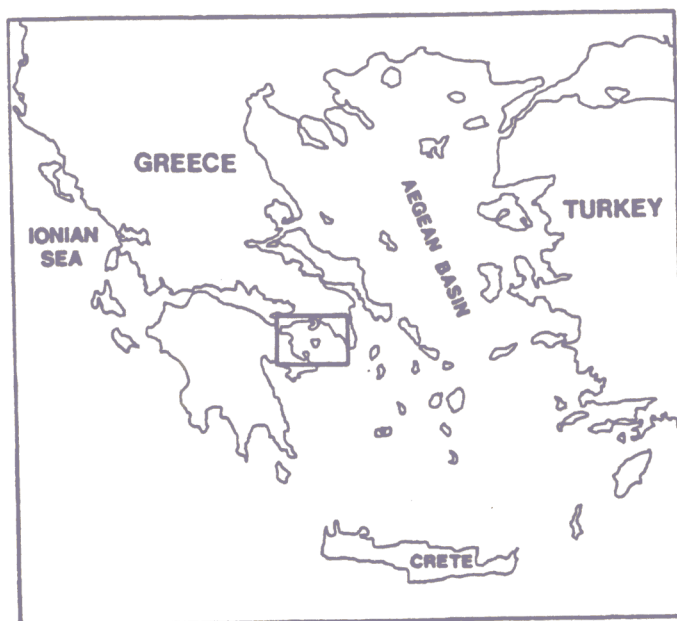
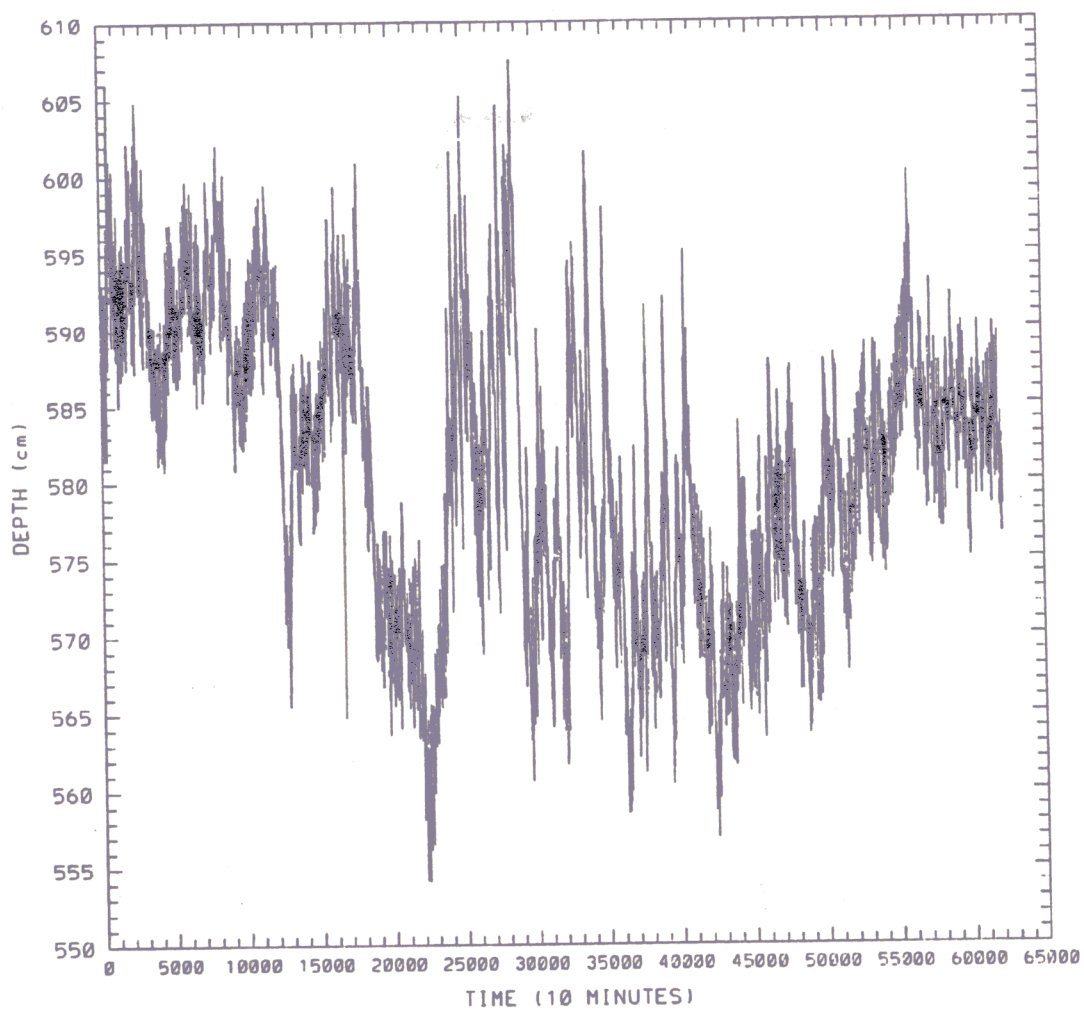
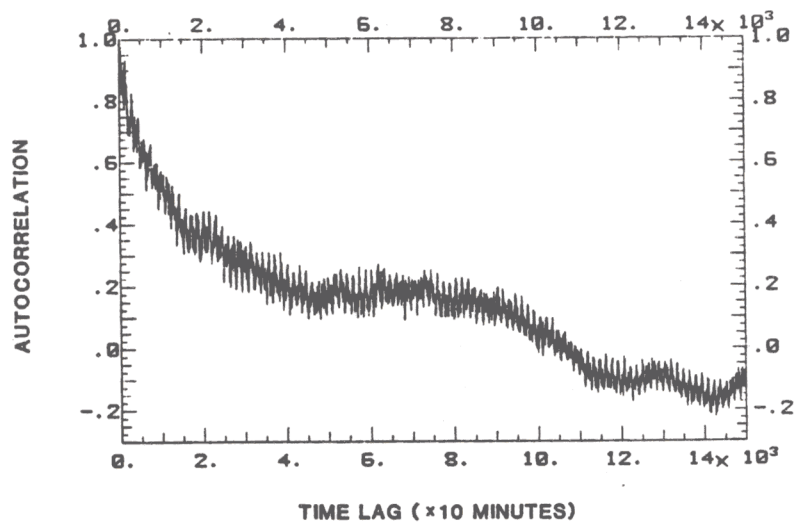


Figure 1. Map of the area. All four stations used in this study are located inside the outlined box of size approximately 60×100 km.

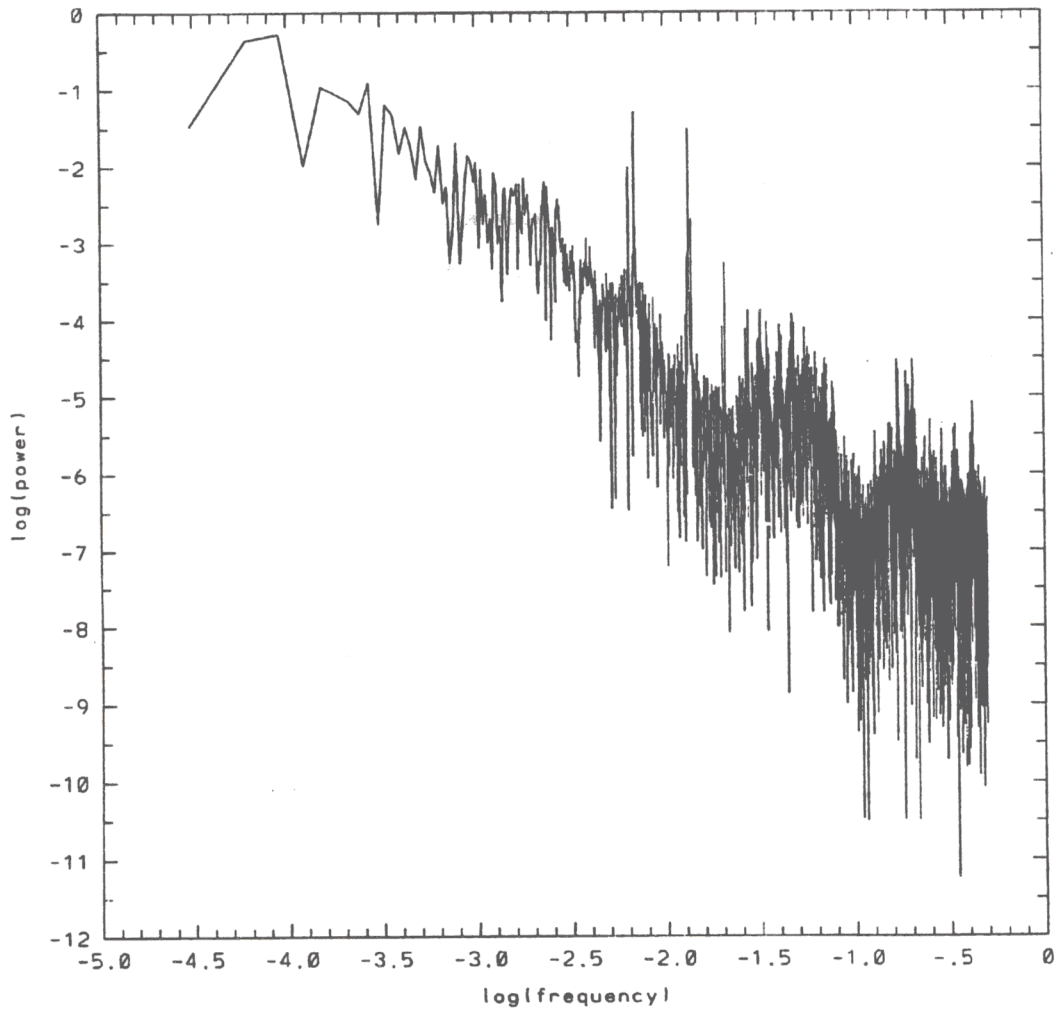


(a) The data represent sea level heights as measured from the bottom of the sea. The temporal resolution is 10 minutes. The record is over fourteen months and it was measured at Salamina station in the Saronikos Gulf of the Aegean Sea, Greece.

Figure 2.



(b) The autocorrelation function of the above data.



(c) The logarithm of the spectral density function. The spectrum shows various peaks on a continuous background, suggesting a nonperiodic evolution.

1. When we constructed the concatenated signal, we had to adjust the individual records in order to deal with the fact that the depth of the bottom of the sea at the four stations is not the same.
2. Even though individual record might show a slight trend the concatenated signal is rather stationary with a trend very close to zero.

Our analysis was carried out for each station individually and when feasible for the concatenated time series. Figure 2 shows the record of one of the stations (Salamina). In the same figure, the autocorrelation function and the spectral density function are also shown. The spectra shows various peaks (periodicities) superimposed on a continuous background. Such spectra often suggest chaotic evolutions. The autocorrelation function decreases with increasing lag. From this graph, we can define a decorrelation time. The decorrelation time may be defined as the lag time at which the autocorrelation falls below a threshold value. This threshold value is not uniquely defined, and in general, it depends on the problem in hand and the assumptions about the data set. In geosciences this threshold value is commonly defined as $1/e$, especially if the autocorrelation function is nearly exponential [23]. But in view of the potential problems when correlated pairs are included in the calculations [18,24], we decided to be more conservative. We thus used a decorrelation time that is more 'restrictive' and at the same time allowed us to work with a sufficiently large number of pairs. We adapted a τ equal to 6000. For lags greater than 6000, the autocorrelation drops and remains below a value of 0.15. From these data, the state vector $X(t)$ was generated (after the trend in the data was removed) and the phase space was produced for embedding dimensions two and higher.

Figure 3 shows the logarithm of the function $C(N, r)$ as a function of $\log r$ for selected embedding dimensions n . The function $C(N, r)$ is defined as:

$$C(N, r) = \frac{2}{N(N-1)} \sum_{i,j=1}^N \mathcal{H}(r - \|x_i - x_j\|),$$

where $\mathcal{H}(x)$ is the *Heaviside* step function. The summation counts the number of pairs (x_i, x_j) for which the distance $\|x_i - x_j\|$ is less than r and which are separated in time by at least the decorrelation time. Therefore, according to equation 2 for a given embedding dimension, this graph will give the exponent d_2 (correlation dimension) once a scaling is identified. For each embedding dimension, in order to identify this scaling region, we plotted the slope: $[\Delta \log C(N, r) / \Delta \log r]$ as a function of $\log C(N, r)$. If there exists a clearly defined scaling region in the $\log C(N, r)$ vs. $\log r$ plots, then we should be able, on a slope vs. $\log C(N, r)$ plot or slope vs. $\log r$ plot, to observe a plateau. This plateau provides an estimation for the exponent d_2 . Figure 4 is a slope vs. $\log C(N, r)$ plot for $n = 10$ and $n = 12$. A plateau is evident over the $\log C(N, r)$ range from about -7.3 to about -5.0 for both embedding dimensions. In both cases, the plateau fluctuates slightly above a value of about 8.8 which is an estimate of the exponent d_2 for both $n = 10$ and $n = 12$. Before, however, we accept these estimates of the exponent d_2 as accurate, we have to present evidence that d_2 has not been underestimated. The reason is that for $D_2 = 8.8$ one would need more than 62,000 points (according to the formula of Nerenberg and Essex [22]). When the number of points involved in the analysis is not large enough, D_2 could be underestimated [8,18]. We, thus, have to repeat the above analysis for a progressively increasing number of points (length of time series) and check whether or not we observe changes in the estimated values of d_2 [4]. As an example, we present Figure 5 which shows the estimated exponent d_2 for $n = 12$ as a function of the number of points available. We clearly see that after about 40,000 points, the value of d_2 does not change. We, thus, can be very confident that the estimated value of d_2 for $n = 12$ from Figure 4 is accurate. When we repeat the above step, for each embedding dimension $2 \leq n \leq 12$, we obtain Figure 6. The scaling exponent d_2 reaches a saturation value of about 8.8 at an embedding dimension $n = 10$.

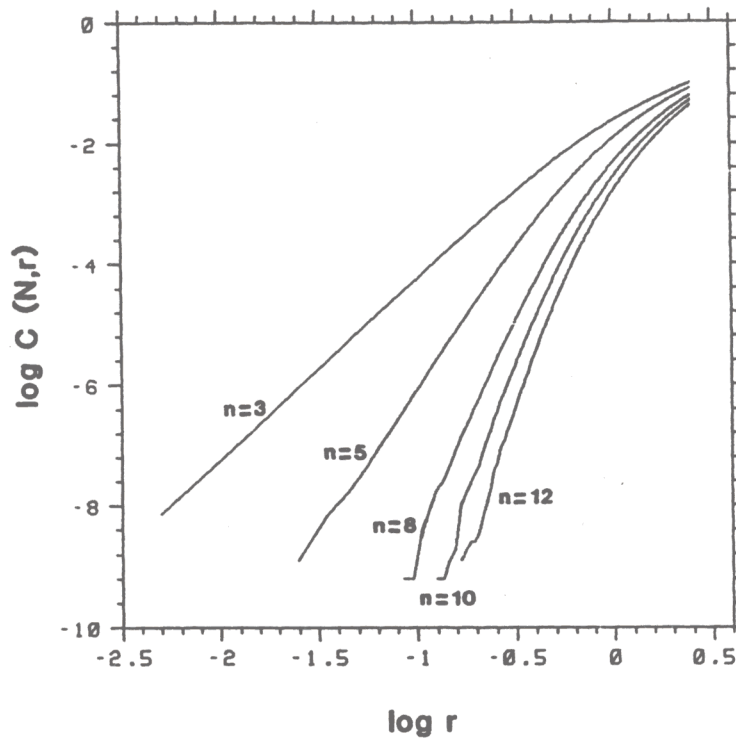


Figure 3. Plot of $\log C(N, r)$ vs. $\log r$ for selected embedding dimensions (n) for the Salamina signal. Note the convergence of slopes as n increases.

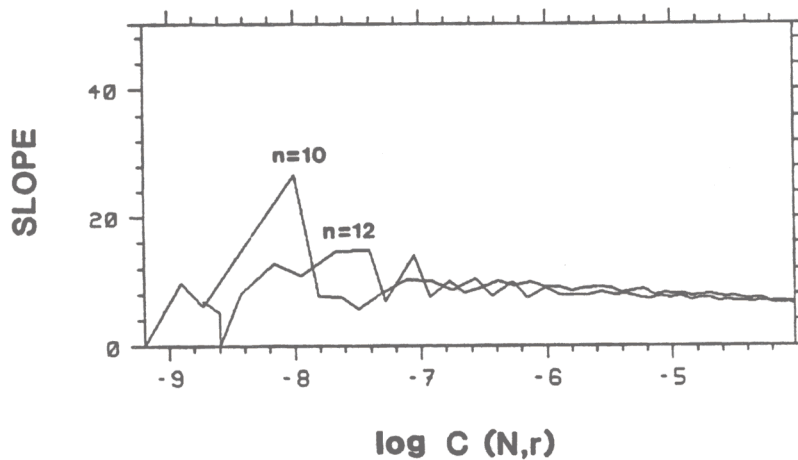


Figure 4. From Figure 3 a plot of slope $[\Delta \log C(N, r) / \Delta \log r]$ vs. $\log C(N, r)$ for embedding dimensions $n = 10$ and $n = 12$. Note the plateau observed at $-7.3 \leq \log C(N, r) \leq -5.0$.

The observed power spectrum of the sea-level data is shown in Figure 2c. One may argue that such a spectra follow a power law of the form $1/f^a$. It has been shown [25] that stochastic processes that exhibit such type spectra will result in a finite correlation dimension $D_2 = 2/a - 1$. In our spectra, depending on the range of frequencies considered, $1.60 \leq a \leq 2.30$. Thus, $1.5 \leq D \leq 3.3$. This dimension is considerably less than our estimated dimension, and thus, we can reject the hypothesis that the sea-level data is such a stochastic process. As an additional test, we repeated the above analysis for a random sample of the same size simulated by inverting the power spectrum, and then, multiplying each complex amplitude by $e^{i\phi}$ where ϕ is uniformly distributed

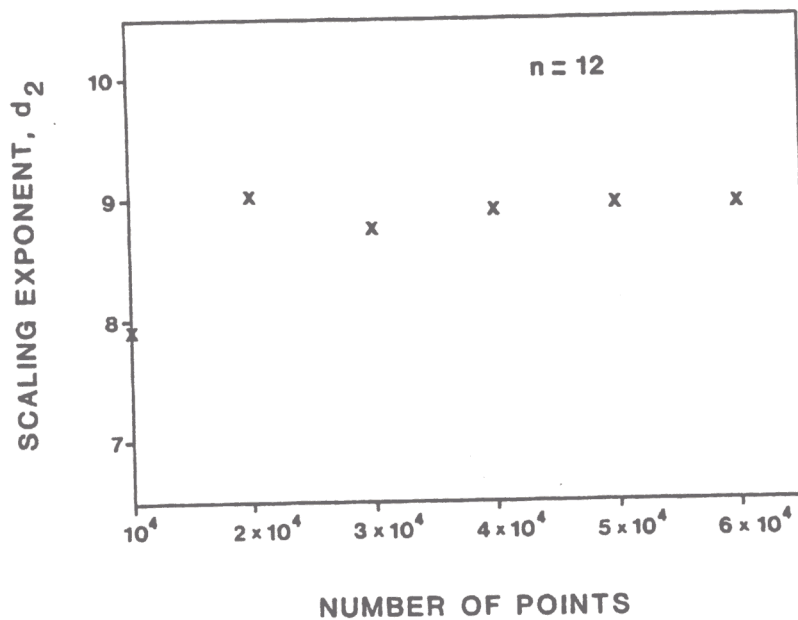


Figure 5. Scaling exponent d_2 (correlation dimension), for the Salamina data as a function of the sample size (length of signal).

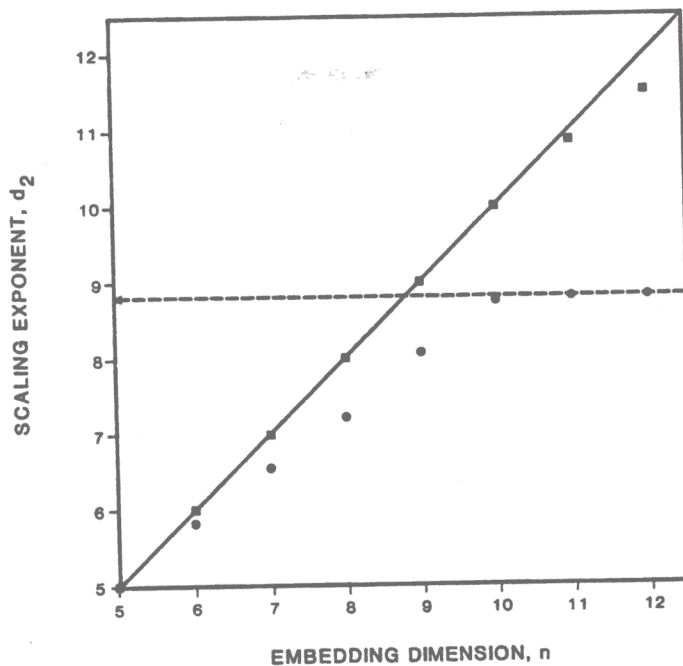
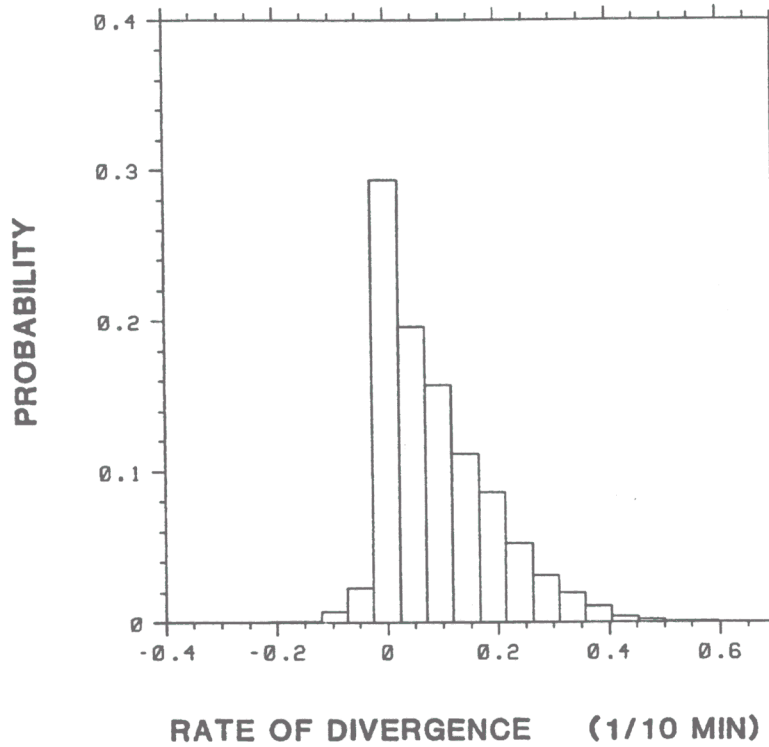


Figure 6. Scaling exponent d_2 , as a function of the embedding dimension n . Full circles correspond to the Salamina signal and squares to a surrogate data set of the same size as the sea-level data. Note the saturation of the scaling exponent observed for the sea-level data while there is no saturation for the surrogate signal. From the third figure, it is estimated that, for the Salamina signal, $D_2 = 8.8$.

in $[0, 2\pi]$. Such a simulation provides a *surrogate* data set [26,27] having the same autocorrelation structure as the real data but not the dynamics. The results are indicated by squares in Figure 6. Note that no saturation for the random sample is observed. Similar conclusions are obtained for all available stations. Specifically, we estimated a correlation dimension of 8.7 for station Souvala, of 9.1 for station Vouliagmeni and of 8.6 for station Ag.Marina. All the dimension calculations made



(a) The largest Lyapunov exponent.

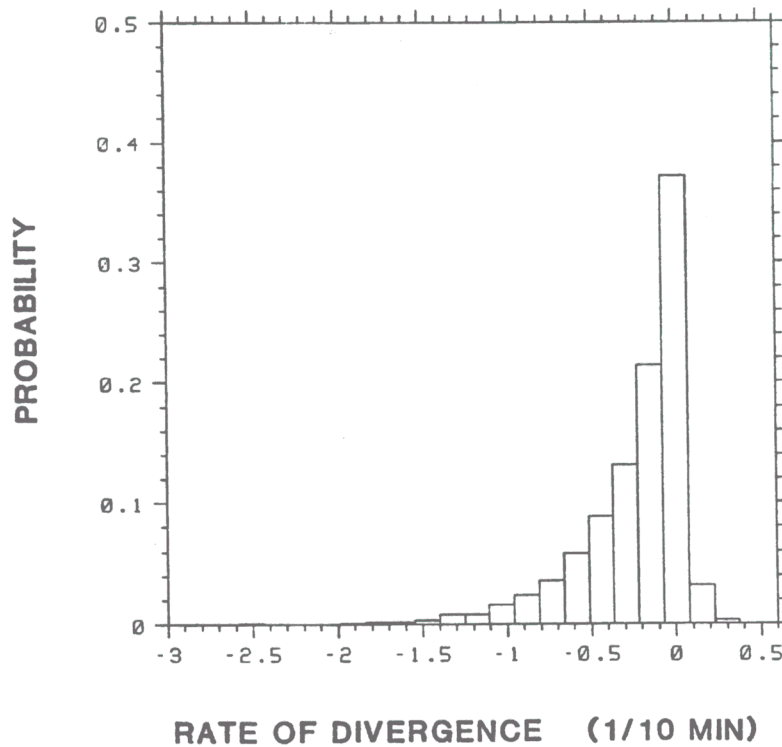
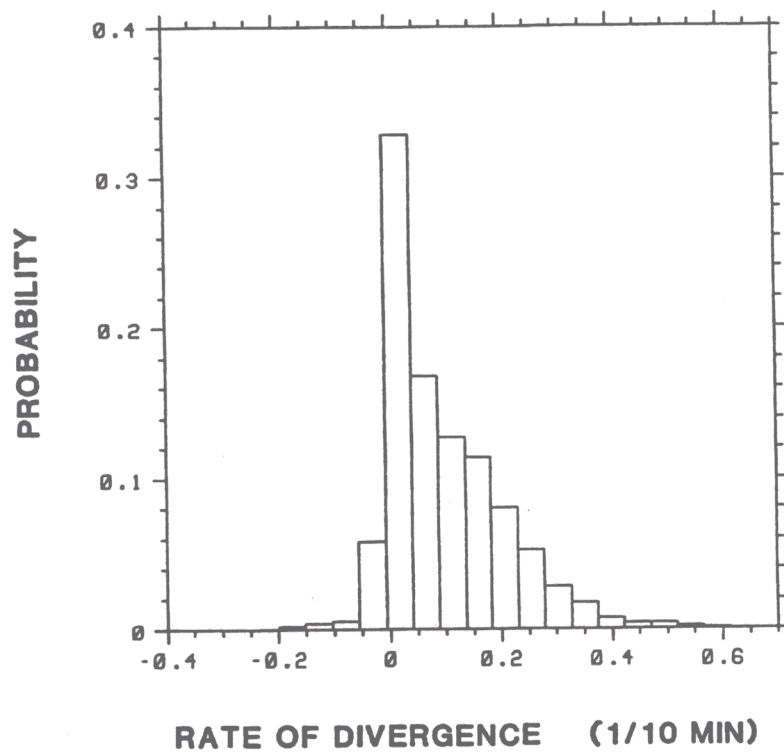
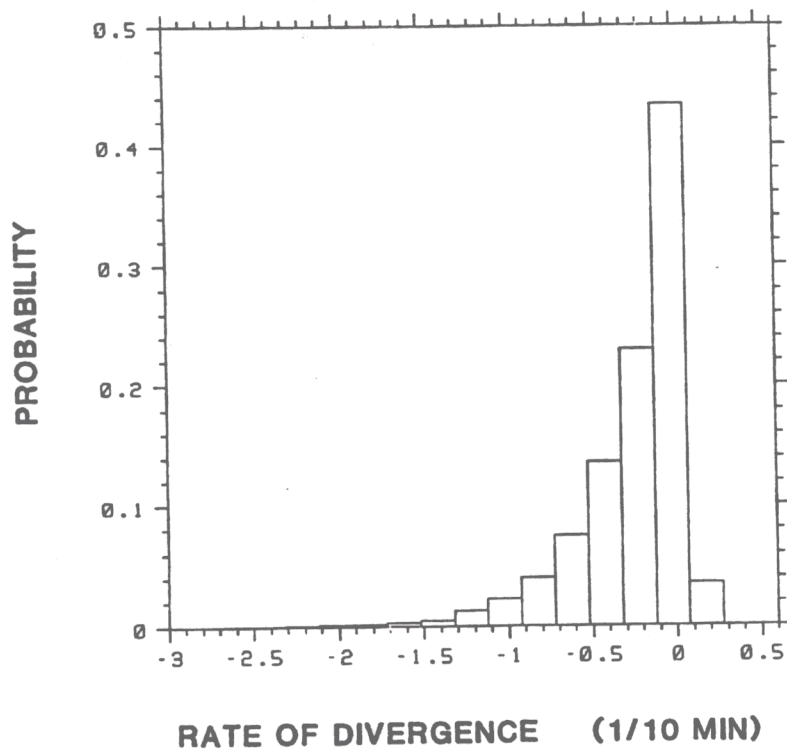
(b) The λ_6 exponent.

Figure 7. Histograms of the probability distribution of rates of divergence whose average yield (a) and (b). This figure corresponds to the Salamina signal.



(a)



(b)

Figure 8. Same as Figure 7 except for the concatenated time series.

use of a fast algorithm developed by Theiler [28]. Nevertheless, for high embedding dimensions and given the size of each record access to supercomputing was necessary. These calculations were performed using the CRAY computer in the National Center for Supercomputing Applications in Urbana, Illinois. It should be mentioned that experimentation with different τ 's ranging from 1000 to 8000 provided dimension estimates of similar magnitudes as above.

The results reported so far strongly suggest that the variability of the marine system in question is dictated by a single low-dimensional attractor (since all estimates are of similar magnitude). The estimated correlation dimensions are not integers suggesting that the attractor is not topological but it is a fractal set. In view, however, of possible inaccuracies in estimating dimensionalities, it is desirable to establish the existence of such attractors with additional evidence. We, therefore, proceed by presenting results dealing with the estimation of the entire spectrum of Lyapunov exponents. The computation of the Lyapunov exponents is based on the work of Sano and Sawada [29] and Eckmann *et al.* [30]. The algorithm at first calls for embedding the data into some dimensional space (preferably of a dimension a little higher than the dimension of the underlying attractor). Then, for a point of the trajectory, we determine the neighbors within a hypersphere of radius r , and subsequently, we seek to construct the operator which describes the evolution of this hypersphere after some time and which provides the rates of divergence along all the directions. Repeating this procedure for a large number of steps (iterations), we can obtain a set of rates of divergence whose averages are nothing else than the Lyapunov exponents we are looking for. Here, we applied their algorithm to each station separately and to the concatenated series. In doing that, we assume that the concatenated series could have been just one observable from our dynamical system. This assumption is more or less justified from the results reported above which indicate that the four stations behave more or less as being parts of a single dynamics encompassing the whole gulf. It can be verified (see also [12]) that the discontinuities of the concatenated signal at the connections between the four individual signals would probably result in an overestimation of the Lyapunov exponents by only about 0.1%. Table 1 gives the results for each of the stations and for all stations together (concatenated record). We choose to present the values of the highest six Lyapunov exponents (all the rest are negative). Together with the mean divergence rate along those six directions, we also report the standard deviation from the mean. This provides us information about the structure of the attractor locally. If the attractor is completely uniform, then the mean rate of divergence along any direction is the same at any place in the attractor (thus the variance is zero). If the variance is not zero, that will indicate that the attractor is highly non-uniform and that the trajectory passes through regions of varying rates of divergence (we may say that the system switches from states of large predictability—low rate—to states of small predictability—high rate). We may observe the following from Table 1:

1. In all cases, we obtained two positive Lyapunov exponents. This is an indication of a chaotic attractor in higher than three dimensions. The fact that their magnitude differs by a factor of three may indicate that the chaotic dynamics arise from the interplay of two independent mechanisms of instability, one of which is more important than the other.
2. In almost all cases, we observe a very close to zero negative value for λ_3 ($-0.013, -0.008, -0.012, -0.006, -0.008$). This is in accordance with the fact that one of the exponents must always be zero expressing the fact that there is no contraction along the direction of the orbit. The fact that the algorithm reproduces this provides confidence about the accuracy of the estimated exponents.
3. There are at least three negative exponents. As the largest positive exponent is about a factor of two greater than the absolute value of the largest negative exponent, it may be that a dominant system time-scale does exist (at least over the data range).
4. The sum of the two largest positive exponents is found between 0.090 and $0.133 (10 \text{ min})^{-1}$. This is an estimate of the Kolmogorov entropy whose inverse (inverse or divergence rate) provides an estimate of the mean predictability time of the signal. Thus, we obtain that

Table 1. The first six largest Lyapunov exponents (mean divergence rate along six directions) and their standard deviation for each station separately and for all stations together (concatenated signal).

i	Mean Divergence Rate $\lambda_i(10 \text{ min})^{-1}$	Variance ^{1/2} $\Delta\lambda_i(10 \text{ min})^{-1}$
Souvala ($N = 61,583$)		
1	0.080	0.093
2	0.037	0.089
3	-0.013	0.099
4	-0.041	0.121
5	-0.107	0.180
6	-0.241	0.328
Ag.Marina ($N = 49,400$)		
1	0.085	0.098
2	0.027	0.092
3	-0.008	0.095
4	-0.048	0.125
5	-0.098	0.167
6	-0.245	0.320
Vouliagmeni ($N = 60,046$)		
1	0.064	0.088
2	0.026	0.081
3	-0.012	0.097
4	-0.053	0.125
5	-0.111	0.181
6	-0.233	0.318
Salamina ($N = 62,184$)		
1	0.091	0.102
2	0.036	0.091
3	-0.006	0.099
4	-0.051	0.123
5	-0.106	0.177
6	-0.237	0.319
All Stations ($N = 233,213$)		
1	0.095	0.106
2	0.038	0.093
3	-0.008	0.101
4	-0.046	0.120
5	-0.105	0.172
6	-0.256	0.348

the signals involved have an intrinsic predictability time of at least one hour but less than two hours. This verifies that our data, which are sampled every 10 minutes, do not suffer from an oversampling problem.

5. All exponents are subject to strong variability (as indicated by the magnitude of the reported standard deviation values). Figures 7 and 8 show histograms of the probability distribution of the rates of divergence whose average yield (a) the largest Lyapunov exponent and (b) the λ_6 exponent, for the Salamina and for the concatenated signal respectively. In all cases, we observe rather broad distributions which are distinctly asymmetric. This is an indication that the underlying attractor is highly nonuniform.

We should mention here that even though in the Lyapunov exponents analysis, we considered the concatenated series, it was not feasible to consider it for the dimensional analysis due to extreme computing demands.

Note that, according to the Kaplan-Yorke conjecture [31], the dimension of the attractor is related to the Lyapunov exponents via the relationship:

$$D = j + \sum_{i=1}^j \frac{\lambda_i}{|\lambda_{j+1}|},$$

where j is defined by the condition that $\sum_{i=1}^j \lambda_i > 0$ and $\sum_{i=1}^{j+1} \lambda_i < 0$. The Lyapunov exponents estimated here will result in values of D around five. This is not exactly what we find from our correlation dimension analysis. Nevertheless, the differences are not great (less than a factor of two). Such differences have been observed elsewhere in real data analysis [12] and quite possibly are caused by how the algorithms process noise in the data.

3. CONCLUSIONS

We have provided strong evidence that the variability of sea-level in a part of the Aegean Sea, Greece can be attributed to a single chaotic dynamical system of a few degrees of freedom with a low-dimensional attractor.

The evidence consists of correlation dimensions and Lyapunov exponents estimates as deduced from observations at four different locations. Correlation dimension estimates are all very similar indicating the existence of a single low-dimensional system. The Lyapunov exponents spectrum includes in all cases two positive exponents indicating that the dynamical system in question is chaotic with its attractor embedded in a higher than three-dimensional space. These positive Lyapunov exponents provide us with a clue about the predictability of such a system which is of the order of one to two hours.

The developments in the study of many chaotic dynamical systems have suggested that nature imposes limits on prediction. At the same time, however, it has been realized that the very existence of the attractors implies that determinism may underlie processes which were thought to be random. We hope that our results will provide a stimulus for further research in this area. It will be interesting, for instance, to investigate the dynamics of models that are used to study and/or predict such phenomena in order to compare the variability described by these models and by the observations.

REFERENCES

1. N.H. Packard, J.P. Crutchfield, J.P. Farmer and R.S. Shaw, Geometry from a time series, *Phys. Rev. Lett.* **45**, 712–716, (1980).
2. D. Ruelle, Chemical kinetics and differentiable dynamical systems, In *Nonlinear Phenomena in Chemical Dynamics*, (Edited by A. Pacault and C. Vidal), Springer-Verlag, Berlin, (1981).
3. F. Takens, Dynamical systems and turbulence, In *Lecture Notes in Mathematics*, Volume 898, Springer, New York, (1981).
4. A.A. Tsonis, *Chaos: From Theory to Applications*, Plenum, New York, (1992).
5. P. Grassberger and I. Procaccia, Characterization of strange attractors, *Phys. Rev. Lett.* **50**, 346–349, (1983).
6. P. Grassberger and I. Procaccia, Measuring the strangeness of strange attractors, *Physica* **9D**, 189–208, (1983).
7. A.A. Tsonis and J.B. Elsner, Chaos strange attractors and weather, *Bull. Amer. Meteor. Soc.* **70**, 16–23, (1989).
8. A.A. Tsonis and J.B. Elsner, Comment on “Dimension analysis of climatic data”, *J. Climate* **3**, 1502–1505, (1990).
9. H.G. Schuster, *Deterministic Chaos*, p. 122, VCH, Weinheim, FDR, (1988).
10. A.A. Tsonis and J.B. Elsner, The weather attractor over very short time scales, *Nature* **333**, 545–547, (1988).
11. A.M. Frazer and H.L. Swinney, Independent coordinates for strange attractors from mutual information, *Phys. Rev. A* **33**, 1134–1140, (1986).
12. C.L. Keppenne and C. Nicolis, Global properties and local structure of the weather attractor over Western Europe, *J. Atmos. Sci.* **46**, 2356–2370, (1989).

13. A.R. Osborne, A.D. Kirwan, Jr., A. Provenzale and L. Bergamasco, A search for Chaotic behavior in large and mesoscale motions in the Pacific Ocean, *Physica* **23D**, 75–83, (1986).
14. A. Provenzale, A.R. Osborne and L. Bergamasco, Lagrangian parcel trajectories in simple deterministic and stochastic models of geophysical relevance, Abstract volume of the General Assembly of the European Geophysics Society, Barcelona, Spain, 13–17, (March 1989).
15. C. Nicolis and G. Nicolis, Is there a climate attractor?, *Nature* **331**, 529–532, (1984).
16. K. Fraedrich, Estimating the dimensions of weather and climate attractors, *J. Atmos. Sci.* **43**, 419–432, (1986).
17. K. Fraedrich, Estimating weather and climate predictability on attractors, *J. Atmos. Sci.* **44**, 722–728, (1987).
18. C. Essex, T. Lookman and M.A.H. Nerenberg, The climate attractor over short time scales, *Nature* **326**, 64–66, (1987).
19. M.B. Sharifi, K.P. Georgakakos and I. Rodriguez-Iturbe, Evidence of deterministic chaos in the pulse of storm rainfall, *J. Atmos. Sci.*, (1990) (to appear).
20. L.A. Smith, Intrinsic limits on dimension calculations, *Physics Lett. A* **133**, 283–288, (1988).
21. L.A. Smith, Computing dimensions from real data. Nonlinear Variability in Geophysics 2, Preprint, pp. 41–42. Paris: Ancienne Ecole Polytechnique, (1988).
22. M.A.H. Nerenberg and C. Essex, Correlation dimension and systematic geometric effects, *Phys. Rev. A* **42**, 7065–7074, (1990).
23. I. Zawadzki, Statistical Properties of Precipitation patterns, *J. Appl. Met.* **12**, 459–472, (1973).
24. P. Grassberger, Do climatic attractor exist?, *Nature* **323**, 609–612, (1986).
25. A.R. Osborne and A. Provenzale, Finite correlation dimension for stochastic systems with power-law spectra, *Phys. D* **35**, 357–381, (1989).
26. J. Theiler, S. Eubank, A. Longtin, B. Galdrikian and J.D. Farmer, Testing for nonlinearity in time series: The method of surrogate data, *Physica D* **58**, 77–94, (1992).
27. J.B. Elsner and A.A. Tsonis, Nonlinear dynamics established in the ENSO, *Geophys. Res. Lett.* **20**, 213–216, (1993).
28. J. Theiler, Efficient algorithm for estimating the correlation dimension from a set of discrete points, *Phys. Rev. A* **36**, 4456–4462, (1987).
29. M. Sano and Y. Sawada, Measurement of the Lyapunov spectrum from a chaotic time series, *Phys. Rev. Lett.* **53**, 1082–1085, (1985).
30. J.P. Eckmann, S. Ollifson-Kamphorst, D. Ruelle and S. Cilberto, Lyapunov exponents from time series, *Phys. Rev.* **34A**, 4971–4979, (1986).
31. P. Frederickson, J. Kaplan, E. Yorke and J. Yorke, The Lyapunov dimension of strange attractors, *J. Diff. Eqs.* **49**, 185–192, (1983).