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Cover picture from a colour transparency by W. S. Pike

Crepuscular rays at Woodlands St Mary, Berkshire, at 2026 BST, 9 August 1987

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CHAOS AND UNPREDICTABILITY OF WEATHER

By A. A. TSONIS

Department of Geosciences, University of Wisconsin-Milwaukee

> Since nothing accidental is prior to the essential neither are accidental causes prior. If, then luck or spontaneity is a cause of the material universe reason and nature are causes before it.
>
> *Aristotle, Metaphysics, Book XI Ch. 8*

WEATHER is a subject that receives a lot of attention. Its variability and diversity has fascinated human beings on this planet since the dawn of human life. Weather and climate are two regulating factors of the living and non-living matter on the planet. Plants, animals and humans are constantly exposed to their continuous action. Under the action of weather conditions that prevailed in the past and of those that prevail today even the surface of our planet has been modified.

It is then only natural that predicting the weather soon became a major task. Despite the advances in our knowledge of the atmosphere however, prediction of the weather is still a puzzle. All of us know that weather cannot be forecasted reliably for a time greater than a few days and for these few days the accuracy is limited. Part of the problem is thought to be the limited representation of the initial stage of the atmosphere. We cannot have an exact forecast unless we can first completely describe the conditions of each particle in the atmosphere. Then the laws of physics can predict the future state of every particle exactly. The above view is referred to as the Laplacian determinism and can be dated back to the late 17th century. Science has come a long way from that view, however. Recent advances in the study of dynamical systems has invalidated the above view. It is the purpose of this article to introduce the reader to these advances which most likely will affect significantly the way we view and try to predict the weather.

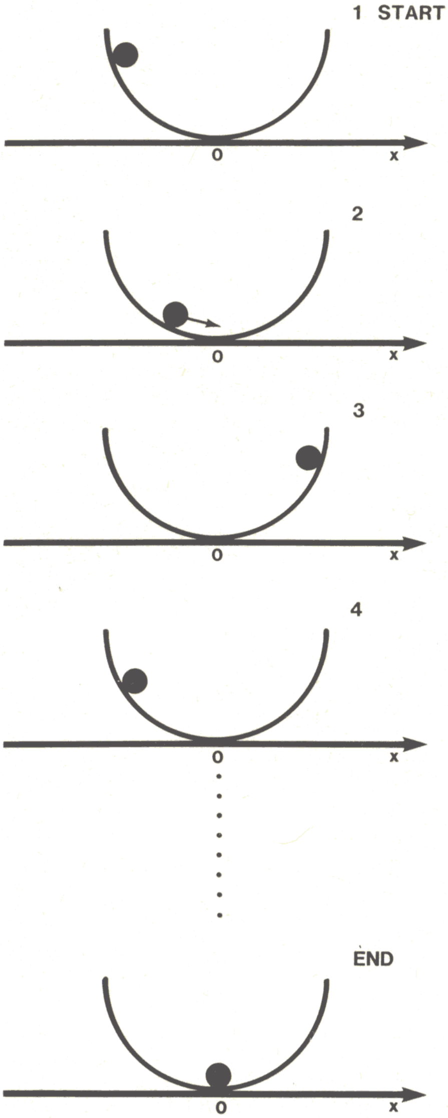
SIMPLE DYNAMICAL SYSTEMS

In the preceding paragraph the term 'dynamical systems' was used. What is a dynamical system? In simple terms a dynamical system is a system whose evolution from some initial state (which we know) can be described by some rule(s). These rules are conveniently expressed as mathematical equations. The evolution of such a system is best described by the so-called state space. Let us first give an example of a simple dynamical system and its state space.

A ball is allowed to roll along a curve from some initial state as it is described in Fig. 1a. This initial state can be completely described by the speed and position of the ball. Since the motion of the ball is, apparently, restricted on a curve the position of the ball at any time can be characterized by its distance from point O on the x-axis, which is taken as the origin. Under such an arrangement, Newtonian physics provide the equation (rules) which describe the evolution of that initial state in time.

Let us assume that to begin with (position 1) the ball is at rest (zero speed) at a given distance from point O on the x-axis. The ball is then let free to roll. As the ball rolls towards point O, its speed increases due to gravity acceleration. Therefore, after a while (position 2) the ball will be closer to point O and will have a higher speed. Once the ball crosses point O its speed decreases because now gravity acts in a direction opposite to its motion. Therefore at some point (position 3) the ball's speed will become zero again. Immediately after that the ball will begin to roll back and after it crosses point O it will once again attain, at some point, a zero speed (position 4). Because there is always some friction, however, the points at which the speed becomes zero (to the right and left of the origin) are not fixed but are found closer and closer to the origin. Finally the ball will come to rest at point O.

Fig. 1(a) (below) A dynamical system is a system whose evolution from some initial state can be determined by some rules. In the figure below the motion of the ball can be completely described by the laws of physics if its initial position and velocity are known.



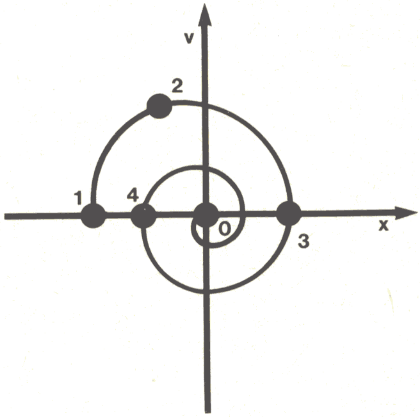


Fig. 1(b) (above) A useful concept in studying the evolution of dynamical systems is the state space. The coordinates of the state space are the necessary variables that are needed to completely describe the evolution of the dynamical system in question. In our example these coordinates are the velocity and position (with respect to point O) of the ball. As the ball rolls back and forth it follows a trajectory in the state space which converges to a fixed point. This point is called an attractor of the dynamical system.

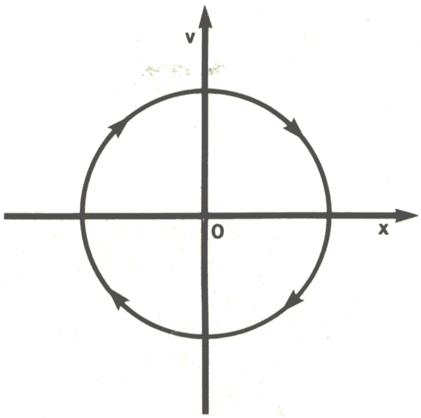


Fig. 2 (above) Another form of an attractor is the limit cycle. In this case all trajectories are attracted by the limit cycle which represents a period evolution. The grandfather clock is a system which possesses a limit cycle as an attractor. Another familiar system with a limit cycle as its attractor is the heart.

Apparently, the evolution of the above dynamical system can be completely described by two variables namely velocity and position (with respect to O). These two variables define the coordinates of the so-called state space. If one plots the velocity (v) as a function of position (x) of the ball one will get Fig. 1b. The solid line is called a trajectory in the state space and apparently describes the evolution of our dynamical system. As it can be seen the trajectory converges to point O. As a matter of fact any other trajectory which will correspond to an evolution of the above dynamical system from a different initial state (velocity and position) will converge to point O (i.e. no matter what the initial state, the ball will always come to rest at point O). The point O in the state space is called an attractor. It 'attracts' all the trajectories in the state space. Apparently, the behaviour of the dynamical system in question can be completely understood. Long term predictability is guaranteed. The evolution of that system can be accurately predicted. The ball will always rest at point O.

So far we have discussed only one form of attractor (i.e. a point). The next simplest form of an attractor is the limit cycle (Fig. 2). A limit cycle in the state space indicates a periodic motion. An example of a system whose attractor is a limit cycle is the grandfather clock where loss of kinetic energy due to friction is compensated mechanically via a mainspring. No matter how the pendulum clock is set swinging a perpetual periodic motion will be achieved. This periodic motion manifests itself in the state space as a limit cycle. Again in the cases of systems which have a limit cycle as an attractor long term predictability is guaranteed.

Another form of an attractor is a torus. The torus looks like the surface of a doughnut (Fig. 3). In this case all the trajectories in the state space are attracted to and remain on that surface. Some electrical dynamical systems have such an attractor. An important characteristic of such an attractor is that any two trajectories which represent the evolution of the system from different initial conditions and which are close to each other when they approach the attracting surface will remain close to each other forever (see Fig. 3). This characteristic can be translated as follows: the two points in the state space where the trajectories enter the attractor can be two measurements (initial states) which differ by some amount. Since these trajectories remain close to each other it means that the states of the system at a later time are going to differ by the same amount that they differed initially. Thus, if we know the evolution of such a system from an initial condition we can predict the evolution of the system from some other initial condition accurately. Again in this case long term predictability is guaranteed.

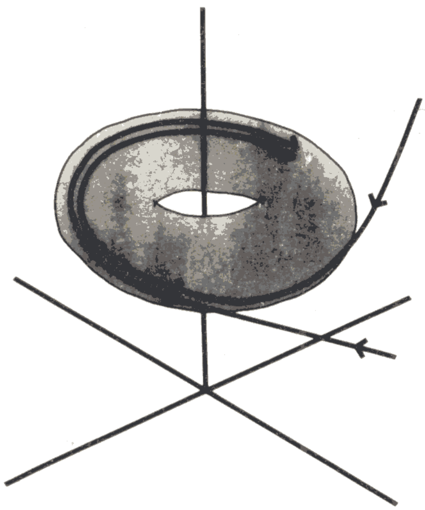


Fig. 3 Another form for an attractor is the torus. In this case the evolution of the corresponding dynamical system from any initial condition will follow a trajectory in the state space which will eventually be attracted and remain forever on the torus. The most important characteristic of a system which exhibits such an attractor is that two initially nearby trajectories on the attractor remain nearby forever.

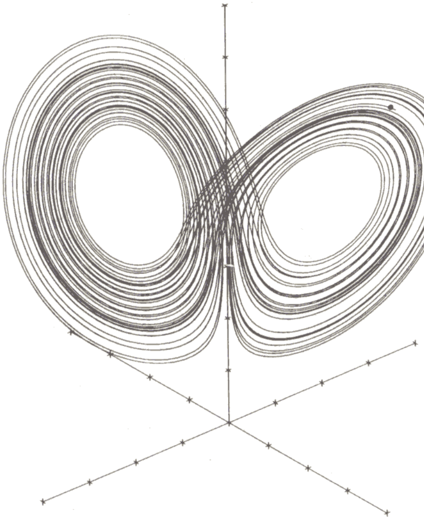
The above-mentioned forms of attractors are 'well behaved' attractors and usually correspond to systems whose evolution is predictable. Often they are called non-chaotic attractors.

ATMOSPHERE: A NOT SO SIMPLE DYNAMICAL SYSTEM

The atmosphere is also a dynamical system and its evolution can be studied with the help of the state space. Of course, to completely describe the evolution of the atmosphere is a very complicated problem. However, simple mathematical models may help to illustrate or understand atmospheric processes. Such a simple model was proposed by Lorenz (1963). That model consists of three simple differential equations that describe the motion of a fluid flow which travels over a heated surface. The warmer fluid formed at the bottom is lighter and it tends to rise creating convection. When Lorenz produced the state space he found out that again in this case all trajectories eventually converge to an attractor which did not look like anything described above. This attractor is shown in Fig. 4a. The most important characteristic of this attractor is that in this case two nearby trajectories do not stay close to each other but they very soon diverge and follow totally different paths in the attractor. That means that the evolution of the system (in this case a simple atmospheric model) from two slightly different initial conditions will be completely different. The above is very effectively demonstrated in Figs. 4a and 4b. The dot in Fig. 4a represents 10000 initial conditions that are so close to each other in the attractor that they are indistinguishable. They may be viewed as 10000 initial atmospheric situations that differ only slightly from each other. If we allow these initial conditions to evolve according to the rules (equations) that describe the system we see (Fig. 4b) that after some time the 10000 dots can be anywhere in the attractor. In other words the state of our system after some time can be anything despite the fact that the initial conditions were very close to each other. In this case we say that our system has generated randomness. We then see that there exist systems which even though described by simple deterministic rules can generate randomness. Randomness generated this way has been termed chaos. These systems are called chaotic dynamical systems and their attractors are called strange or chaotic attractors.

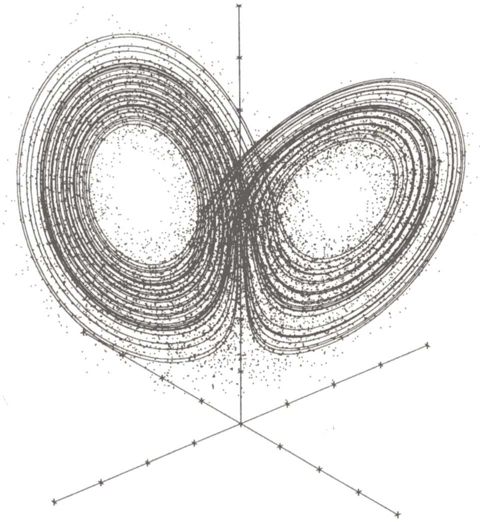
The implications of such findings are profound. If one knows exactly the initial conditions one can follow the trajectory that corresponds to the evolution of the system from those initial conditions and basically predict the evolution forever. The problem, however, is that we cannot have a *perfect* measurement. Our instruments can only measure *approximately* the various parameters (temperature, pressure, etc.) which will be used as initial conditions. Therefore, there is always going to be some deviation of the measured from the actual initial conditions. They may be very close to each other but they will not be the same. In such a case, even if we completely know the physical laws that govern our system, due to the action of the underlying attractor the state of the system at a later time can be totally different from the one predicted. Simply due to the nature of the system, initial errors are amplified and its prediction power is very limited. Therefore, we see that the existence of a strange or chaotic attractor coupled with the fact that we can only know an initial situation approximately imposes, naturally, prediction limits to the system.

The above conclusions are obtained from the study of simple dynamical systems whose mathematical formulation is exactly known. In the absence of a mathematical formulation of a dynamical system, the dynamics can be inferred from a single record of an observable variable $X(t)$ from that system. The physics behind such an approach is that a single record from a dynamical system is the outcome of all the interacting variables and thus information about the dynamics of that system should be included in any observable variable. The mathematical procedure to prove the existence and nature (chaotic or not) of an attractor is rather elaborate and beyond the scope of this paper. Those that are interested for more details on the mathematics should refer to Packard *et al.* (1980), Takens (1981), Rouelle (1981) and Tsonis and Elsner (1988). Returning to



By courtesy of Dr James Crutchfield

Fig. 4 (a) An example of a strange attractor with implications in the forecasting the weather problem. This structure in the state space represents the attractor of a fluid flow which travels over a heated surface. All trajectories (which will represent the evolution of that system for different initial conditions) will eventually converge and remain on that structure. However, any two initially nearby trajectories in the attractor do not remain nearby but they diverge.



By courtesy of Dr James Crutchfield

Fig. 4 (b) The effect of the divergence of initially nearby trajectories in the attractor: The dot in Fig. 4 (a) represents 10000 measurements (initial conditions) which are so very close to each other that they are practically indistinguishable. If we allow each one of these states to evolve according to the rules, because their trajectories diverge irregularly, after a while their states can be practically anywhere.

our subject the atmosphere is obviously far more complicated than the simple system investigated by Lorenz. Because of that, complete and/or exact mathematical formulation of the atmospheric processes has not yet been developed. Soon, therefore, observable weather variables were considered in the search for attractors in weather and climate. Nicolis and Nicolis (1984) used oxygen isotope records of deep-sea cores spanning the past million years. These data are related to temperature fluctuations during that time interval. Fraedrich (1986) used daily pressure data over a period of 15 years and Essex *et al.* (1987) used daily geopotential data over the last 40 years. Lately, Tsonis and Elsner (1988) extended the analysis to very short time scales using 10 second averages of vertical wind velocity over 11 hours. In each one of the above studies it has been concluded that an attractor is present. It should be mentioned, however, that these studies do not necessarily suggest that there exists a unique attractor at all time scales. It may be that the attractors (and thus predictability) are different at different time scales or it may be that when considering a certain time scale we are only looking at a certain part of a grand attractor. Both possibilities are very exciting. It is anticipated that research on this area will provide many clues about the interactions between different time scales.

CONCLUSION

Many systems in nature are chaotic. The developments in the study of chaotic dynamical systems have suggested that nature imposes limits in prediction. At the same time, however, it has been realized that the very existence of the attractors implies that randomness is restricted to the attractors. The atmosphere may be chaotic but its evolution is confined in a specific area in the state space which is occupied by the attractor. No states outside this area are allowed. The winds, for example, associated with a high pressure system can never be blown counterclockwise. Something more intense than a hurricane cannot exist.

The theory of chaotic dynamical systems has improved our understanding of the behaviour of the atmosphere. At the same time, even though it has provided an excuse for the unpredictability of weather, the theory of dynamical systems is slowly shaping up our way of investigating the weather and its prediction. For example, it may very well be that generalizations based on the study of specific cases (which may never happen exactly again) can no longer be appropriate.

Together with some pessimism the study of chaotic dynamical systems provides some optimism. We may never be able to predict the weather exactly but improvements in weather forecasting are feasible if we improve the completeness and accuracy with which we measure the initial condition of the atmosphere.

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