
A Proposal for a New Statistic and Technique Development for the Design and Evaluation of Cloud Seeding Experiments

A.A. Tsonis¹, G.L. Austin and S. Lovejoy
McGill Weather Radar Observatory
Ste-Anne-de-Bellevue, Québec

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ABSTRACT *The use of short-term predictions of rain flux from isolated showers is discussed in the context of the design and evaluation of cloud-seeding experiments. It is found that for a sample of 85 seeded convective clouds a seeding effect of 25% could be detected at the 10% significance level.*

RÉSUMÉ *L'utilité des prédictions à court terme de la pluie qui tombe en averses isolées est discutée dans le contexte de la planification et de l'évaluation d'un projet d'ensemencement de nuages. On a trouvé que pour les nuages convectifs, un effet d'ensemencement de 25% pourrait être détecté au niveau significatif 10% avec 85 expériences.*

1 Introduction

One of the most important problems encountered in the design and evaluation of a cloud-seeding experiment is the natural variability of rainfall from one experimental unit (i.e. cloud) to the next. When the experiment utilizes a rain-gauge network, an additional problem is the network sampling variance (McGuinness, 1963). However, if the gauge density of the network is reasonably high, the latter source of variability is not as important as cloud variability (Silverman et al., 1981). When a weather radar is used, sampling variability is insignificant.

Total sample size requirements for detecting seeding effects, as well as for evaluation procedures, depend to a great extent on the statistic used in evaluating the outcome of an experiment. A common procedure makes use of a transformation of the total rain amount, say R . Examples are $\log R$ (Biondini, 1976) and $R^{\frac{1}{2}}$ (Simpson, 1972). Although the distributions of these transformed variables fit (unseeded) data quite well over the bulk of observed histograms, the gamma ($R^{\frac{1}{2}}$) and lognormal ($\log R$) distributions retain long tails. Thus, they have low power and consequently require

¹Present affiliation: Atmospheric Environment Service, Downsview, Ontario.

rather long experimental periods for the detection of moderate seeding effects. The power of cloud-seeding experiments might be enhanced through the use of covariates, such as cloud stability, lifted cloud-top temperatures, and echo-top height (Vardiman and Moore, 1978; Rottner et al., 1980). Unfortunately, none of the covariates considered seems to have been established as having significant impact in this regard. The purpose of the present study is to present a new statistic and develop a technique for the evaluation of seeding experiments that has a greater likelihood of detecting a seeding effect, for any given experimental design, and thus requires fewer experimental units to detect the seeding effect at any level and for any preselected probability of detection.

We use radar data to illustrate our analysis, although, as it will become clear later, the same analysis could be carried out for a reasonably dense gauge network. It is recognized that seeding may modify the drop size spectrum thus introducing difficulties in the evaluation procedure. However, there is no evidence that such a modification would mask a real seeding effect. For the purpose of this work, radar rainfall measurement introduces some advantages. Rain cells can be followed in time, and records of their behaviour are available for detailed examination.

2 Data base

The analysis described here is based on weather radar images of convective cells from the GATE experiment in the Tropical Atlantic in 1974, and data recorded in 1980 from the MRL-5 radar in Spain during the Precipitation Enhancement Project (PEP) experiment. A computer program was used to identify, isolate and calculate the area and flux of rain (area \times average rainfall rate) of each cell from the digital magnetic tapes and to print out PPIs in which individual cells are identified by a cell number. The spatial resolution for the GATE data was $4 \text{ km} \times 4 \text{ km}$, and for PEP data, $2 \text{ km} \times 2 \text{ km}$. The temporal resolution was 5 min for all of the data sets, which allowed individual cells to be followed in time. In this study "cell" means any isolated patch of rain appearing as an echo on the radar maps, regardless of its size.

Cells were selected by following their history through the map sequence. Each cell was followed from the beginning of its "radar life" up to its end, its flux and area being noted at every time step (5 min). Cells that existed when the radar was switched on or that moved into radar range are considered in the sample as well, since these situations represent real occurrences in a radar data set. During their lifetimes cells can merge with other cells or split; it is a matter of arbitrary definition as to how merging or splitting is regarded. For this study the following procedure was adopted:

- 1 If a "parent" cell splits and the two (or more) "daughter" cells continue, separated on the radar map by a distance of at least four grid points, then the parent ends and the new cells begin their lifetimes.
- 2 If a parent cell splits and the daughter cells later rejoin, then the parent is assumed to continue its lifetime, ignoring the birth of the daughters.
- 3 If two (or more) cells exist separately for 15 min or more and then merge and remain merged, a new cell is considered to have started its lifetime, while the two (or more) original cells are considered to have ended theirs.

- 4 If two cells exist separately, merge and separate then the merging is disregarded. The flux of the cells while merged is found by linearly interpolating the fluxes before and after the merging.

The cells included in the sample are long-lasting cells, with lifetimes greater than 100 min. It is understood that for a real experiment, revisions would be needed to redefine the control subset of the original data. Even though cloud lifetime is very important, not much research has been performed in this field. Therefore, there is insufficient knowledge connecting "seedability" and cloud lifetime. Schemenauer and Isaac (1980) state that a cell should last at least 20 min after seeding (see also Isaac et al., 1982) in order for the ice crystals generated to grow to precipitable sizes and fall out of the cloud. In a more recent study, Schemenauer and Isaac (1984) conclude that clouds with cloud-top lifetimes of about 20 min may have the potential to be seeded. Dennis (1980) also recognizes the importance of longer-lasting cells in a weather modification project. In general, one should have enough time to decide whether or not a cell will be seeded, to send the airplane, to perform the seeding and to follow the cell for a reasonably long period. Thus, the use of longer-lasting cells is justified from the practical point of view. The above, however, does not support the use of cells lasting 100 min or so, and that is not our purpose. The data used in this work merely illustrate the proposed method. The idea behind this paper was the use of rain-flux predictions in the design and evaluation of a cloud-seeding experiment. In order to evaluate various predictive schemes, the use of longer-lasting cells was necessary (Tsonis and Austin, 1981). In a more general approach, however, the restriction of using long-lasting cells can be removed.

3 Statistic used

Suppose that at time T a cell is seeded and that, before seeding, a technique is available that allows exact knowledge of what the flux of this cell will be at time $T + \Delta t$, say, for $\Delta t = 30$ min. Then even with one seeding experiment it would be possible to know whether or not seeding has any effect. Naturally, the perfect forecast technique does not exist. However, the observed distribution of errors in the forecast technique can be used as a statistical means to test for seeding effects. The feasibility of using predictions of rainfall patterns in the verification of cloud seeding was first discussed by Nason and Lopez (1967). Their method involves fitting a plane or some higher order mathematical surface to the control area data and extending this surface through the target area. The target predictions represent the rainfall expected with no seeding; and these values are then compared with measured amounts to determine the seeding effects. This was done using a rain-gauge network at the control target area; it was concluded that the method is not acceptable for convective type precipitation, mainly because of its great spatial variability.

In this section use will be made of the results presented by Tsonis and Austin (1981) concerning errors in the forecasting of rain flux. It was shown there that the assumption of steady state with translation produces forecasts with an accuracy as high as that of more elaborate extrapolation schemes and it will be used for the analysis described below. It should be noted that any other forecast scheme of

comparable or better accuracy could also be used. The statistic proposed and used in this study is defined as follows:

$$y = \frac{F_f - F_0}{\max(F_f, F_0)} \quad (1)$$

where F_f is the forecast total flux of the cell and F_0 is the observed total flux. Clearly the distribution of y will be bounded by -1 and $+1$. According to our method of forecasting, we have:

$$F_f = F_t \quad \text{and} \quad F_0 = F_{t+\Delta t}$$

Therefore, $F_f - F_0$ is a measure of the change in the total flux of a cell in the time interval Δt . We may choose $\Delta t = 30$ min, for example, so that

$$x = -y = \frac{F_{t+30} - F_t}{\max(F_{t+30}, F_t)} \quad (2)$$

and denote the density function of this statistic by $f(x)$. The x statistic will suffer to the same extent as the transformations $R^{\frac{1}{2}}$, $\log R$, etc., if the assumption of a multiplicative seeding effect is not valid. In such a case, transformations (especially the non-linear ones) may distort treatment effects (Dennis, 1980). However, it precludes the impact of a long-tailed distribution on the power of a statistical test for seeding effect. The choice of $\Delta t = 30$ min in (2) does not imply that we expect that any effect will be detectable after 30 min. This interval was chosen for demonstration purposes only. The effect of choosing another interval will be discussed later.

4 Growing and decaying cells – GATE data set

The cells that are crossing a given area are in either a growing or a decaying stage. That means that the distribution of x will contain information from growing as well as from decaying cells. Accordingly, the distribution can be considered a mixture of two other distributions, say, with density functions $d(x)$ and $g(x)$, where d stands for decaying and g for growing. From the point of view of the design and evaluation of a weather modification experiment, this may not be desirable since it might be that seeding modifies growing and decaying cells in different ways, simply because precipitation enhancement is closely connected with natural precipitation mechanisms and cloud microphysics, which are different in growing and mature clouds. Commenting on this point, Dennis (1980) suggests that only growing clouds should be seeded. However, even though there is not much scientific evidence about how seeding acts in different cloud stages, the stratification of the data into growing and decaying cells would probably be desirable in any experimental design. Most of the cells in the sample were clearly in either the decaying or growing stage, although a few initially grew and then decayed. For these cases, the cells were categorized as either growing or decaying depending on the predominant phase. In a real-time operation the classification into growing and decaying cells can be possibly automatically achieved by the measurement of the vertical reflectivity profile of the cell (Massambani, 1982). For these reasons, decaying and growing cells are considered separately.

In presenting the method we take the following steps in the order given:

- 1 Selection of the samples (i.e. growing and decaying) for the control cases.
- 2 Determination of the parameters of the distributions for the control cases.
- 3 The generation of a simulated data set (real data + assumed seeding effect).
- 4 Determination of the parameters of the distribution for the seeded data.
- 5 Determination of the necessary number of experiments (i.e. seeded cases) in order to detect a seeding factor at specified confidence levels.

Steps 1 and 2 are described in Tsonis (1982) where statistical procedures are presented for the estimates of μ_g , σ_g , and μ_d , σ_d . Here μ_g and μ_d denote the means of the distributions for the growing and decaying cells, respectively, and σ_g and σ_d their corresponding standard deviations. For the control samples, 100 values of x for the decaying cases and 97 for the growing cases were measured. The estimation procedures demonstrate that the sample means and standard deviations remain practically unchanged for sample sizes greater than 40. In this way, good estimates of μ_g , μ_d , σ_g and σ_d were obtained. The estimated values are:

$$\begin{aligned}\mu_g &= 0.12 & \sigma_g &= 0.37 \\ \mu_d &= -0.20 & \sigma_d &= 0.36\end{aligned}$$

It is useful to recall that the two distributions (for growing and decaying cases) are not antisymmetric. The reason is that the fluctuations of the flux in time are generally greater and more frequent in a growing stage than in a decaying one; i.e. the chance of getting a negative value of x from a growing cell is higher than that of obtaining a positive value of x from a decaying cell. Therefore, the absolute value of μ_g will be less than the absolute value of μ_d . Moreover, according to the above, the standard deviation σ_d should be expected to be slightly less than σ_g . The estimated values of μ_g , μ_d , σ_d and σ_g are fully consistent with these expectations.

5 Determination of the number of experiments as a function of the seeding factor

a Growing Cells

The distribution of x for the growing cases, as defined, is to be bounded at both sides, i.e. at -1 and $+1$. Even though this distribution is bounded and is relatively narrow compared to other distributions used in weather modification experiments (log-normal, gamma, etc.), it is very sensitive to seeding effects. Figure 1 shows a comparison of the frequency histogram of the unseeded (control) cases with a frequency histogram of the simulated cases when the multiplicative seeding factor $r = 1.4$ (40% increase) is assumed. It is evident that for increasing r more positive values are produced and the distribution for the seeded cases narrows and shifts towards $+1$. Obviously, when negative seeding effects are assumed, the distribution will shift towards -1 as r approaches 0.0.

Therefore, the distributions of the seeded cases will be dependent on r , each with a mean and a standard deviation that are functions of r , namely, μ_{gr} and σ_{gr} , respectively.

For a multiplicative seeding factor r , if the exact distribution $g(x)$ is known, it

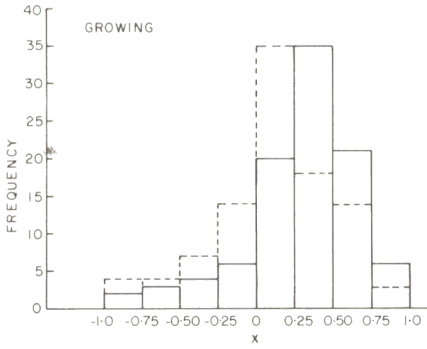


Fig. 1 Simulated frequency histogram of x for the growing cells when a seeding effect of 40% is assumed (solid lines), compared with the control frequency histogram (dashed lines).

would be possible for μ_{gr} and σ_{gr} to be defined analytically. However, it is possible to seek an expression for $\mu_{gr} - \mu_g$ as a function of the seeding factor r , if r is assumed to be a constant multiplicative factor acting, say, 30 min after seeding. In this case, one would simulate values of \bar{x} and \bar{x}_r from the control sample where \bar{x} is the sample mean value of x for the unseeded cases and \bar{x}_r is the sample mean value of x when a multiplicative seeding factor r is assumed; i.e.

$$\bar{x} = \frac{\sum_{i=1}^{n_1} \frac{(F_{t+30})_i - (F_t)_i}{(F_{t+30})_i} + \sum_{i=1}^{n_2} \frac{(F_{t+30})_i - (F_t)_i}{(F_t)_i}}{n_1 + n_2}$$

or

$$\bar{x} = \frac{n_2 \left\langle \frac{F_{t+30}}{F_t} \right\rangle - n_1 \left\langle \frac{F_t}{F_{t+30}} \right\rangle + n_1 - n_2}{n_1 + n_2}$$

and

$$\bar{x}_r = \frac{n_2' \left\langle \frac{F_{t+30} \cdot r}{F_t} \right\rangle - n_1' \left\langle \frac{F_t}{F_{t+30} \cdot r} \right\rangle + n_1' - n_2'}{n_1' + n_2'}$$

where

$$\langle \rangle \text{ indicates } (1/m) \sum_{i=1}^m \text{ and}$$

n_1 and n_2 are the number of positive and negative values of x in the unseeded sample and n_1' and n_2' are the corresponding values of x when a seeding factor r is assumed; for $r > 1$, $n_1' > n_1$ and $n_2' < n_2$ (i.e. more positive values are produced while $n_1 + n_2 = n_1' + n_2'$). For the population of the growing cells, $\bar{x}_r - \bar{x}$ will be an estimate of $\mu_{gr} - \mu_g$. In other words, $\mu_{gr} - \mu_g$ will represent the difference between the means of the seeded and unseeded populations for a certain seeding factor r .

The results from the simulations are very interesting in that they show that for a given value of r , $\bar{x}_r - \bar{x}$ varies very little from some mean value as n increases (at the

TABLE 1. Typical results of the procedure for estimating $\mu_{gr} - \mu_g$ as a function of the seeding factor r .

Seeding Factor r	Sample Size n	$\bar{x}_r - \bar{x}$	Mean Value
1.1	23	0.069	0.067
	41	0.068	
	58	0.067	
	80	0.067	
	97	0.064	
1.4	23	0.223	0.218
	41	0.216	
	58	0.221	
	80	0.217	
	97	0.212	
2.0	23	0.409	0.397
	41	0.389	
	58	0.403	
	80	0.396	
	97	0.388	
2.6	23	0.517	0.500
	41	0.486	
	58	0.506	
	80	0.498	
	97	0.490	
3.8	23	0.635	0.611
	41	0.591	
	58	0.616	
	80	0.611	
	97	0.602	

most $\pm 3\%$). Typical results from these simulations for the growing cases are shown in Table 1. Since $\bar{x} \rightarrow \mu_g$, and $\bar{x}_r \rightarrow \mu_{gr}$ as n increases, the mean value of $\bar{x}_r - \bar{x}$ for each value of r can be assumed to be a good estimate of $\mu_{gr} - \mu_g$ for that r . In this way a graph of $\mu_{gr} - \mu_g$ versus seeding factor can be constructed. The graph is shown in Fig. 2 where it can be seen that a smooth curve fits the values of $\mu_{gr} - \mu_g$. (The solid curve corresponds to the growing cases and the dashed curve to the decaying cases.)

Similarly, an expression for $\sigma_g - \sigma_{gr}$ can be derived. After simulations along similar lines, Fig. 3 shows $\sigma_g - \sigma_{gr}$ as a function of the seeding factor. The shape of the curve appears somewhat unusual, but can be understood through the following reasoning. Consider, for example, the solid curve, which corresponds to the growing cells. For $r > 1$, one should expect that σ_{gr} decreases and therefore $\sigma_g - \sigma_{gr}$ approaches the value of σ_g asymptotically (i.e. 0.37). For $r < 1$, more negative values are generated, so that initially the spread, σ_{gr} , will increase and $\sigma_g - \sigma_{gr}$ will become zero once more. Subsequently, σ_{gr} will start to decrease and $\sigma_g - \sigma_{gr}$ will increase attaining a value of 0.37 for $r = 0.0$. Similar arguments can be made for the dashed curve, which corresponds to the decaying cells. The curves of Figs 2 and 3 will be used for the subsequent calculations.

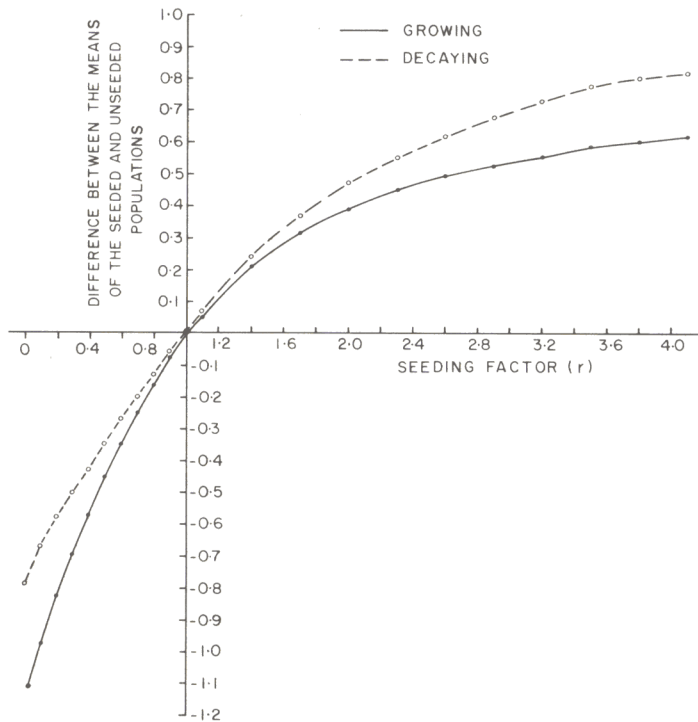


Fig. 2 Difference between the means of seeded and unseeded populations as a function of the seeding factor r .

One can now proceed to the method of determining the number of seeded cases (n), which will be needed to detect the seeding factors at specified significance levels. No assumptions will be made about the exact distribution of x for the seeded or unseeded cases. The well-known values of the means and standard deviations of the populations of the seeded and unseeded cases will be used together with the central limit theorem for the distribution of the mean. This implies that: a) the mean \bar{x} of a random sample of size n taken from the population of the unseeded cases will be asymptotically normally distributed with mean μ_g and standard deviation σ_g/\sqrt{n} ; and b) that the mean \bar{x}_r of a random sample of size n taken from a seeded population of a certain r will be asymptotically normally distributed with mean μ_{gr} and standard deviation σ_{gr}/\sqrt{n} . As shown in Tsonis (1982) the asymptotic distribution may be used even when n is as small as 5. Such a consideration can be presented graphically as in Fig. 4, which shows the distributions of the sample means for the unseeded cases and the seeded cases with a seeding factor $r_1 > 1$. Note that $N(\mu_{gr_1}, \sigma_{gr_1}/\sqrt{n})$ is narrower than $N(\mu_g, \sigma_g/\sqrt{n})$, since $\sigma_{gr_1} < \sigma_g$ for the seeded cases. It should be noted at this point that since the mean and standard deviation for the unseeded cases can be accurately estimated, there is no need to consider an unseeded sample in developing the technique. Therefore, we do not concern ourselves with tests based on the difference between the means of seeded and unseeded samples. We are interested in

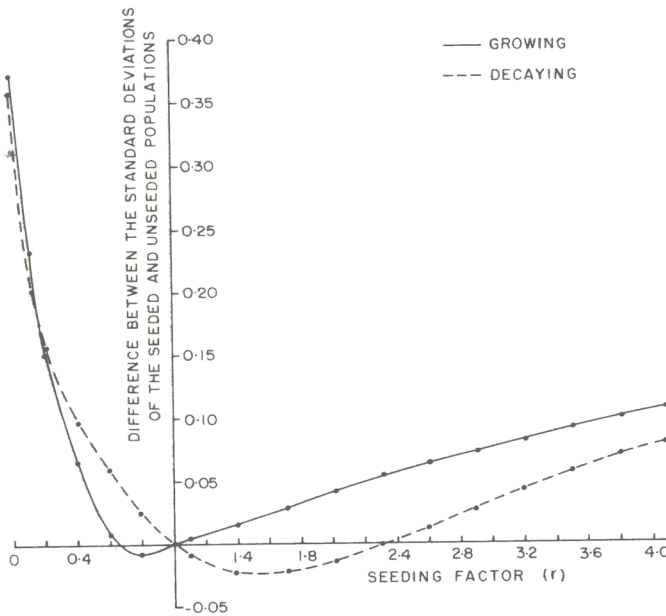


Fig. 3 Difference between the standard deviations of the unseeded and seeded populations as a function of the seeding factor r .

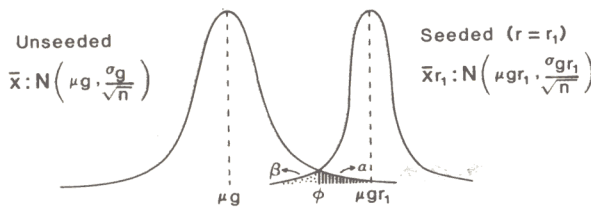


Fig. 4 Illustration of the distributions of the sample means for the unseeded cases and the seeded (for $r = r_1 > 1$) cases.

determining the number of the seeded cases, n , such that the distributions of the means (see Fig. 4) are narrowed at the point that we can test, at specified significance levels, whether the seeded sample comes from the unseeded population or from a seeded population.

Suppose that n cells were seeded and a value for the sample mean of the statistic x was calculated and found to be ϕ . The probability that the sample mean \bar{x} for the unseeded cases is greater than ϕ , is given by:

$$P(\bar{x} \geq \phi) = \int_{\phi}^{\infty} \frac{\sqrt{n}}{\sqrt{2\pi}\sigma_g} \exp \left[-(\bar{x} - \mu_g)^2 n / 2\sigma_g^2 \right] d\bar{x}$$

or

$$P(\bar{x} \geq \phi) = 1 - Z \left(\frac{\phi - \mu_g}{\sigma_g / \sqrt{n}} \right) \quad (3)$$

where:

$$Z(t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^t e^{-y^2/2} dy$$

Similarly, the probability that the sample mean \bar{x}_r of any sample from the seeded population is greater than ϕ is given by:

$$P(\bar{x}_r \geq \phi) = 1 - Z\left(\frac{\phi - \mu_{gr}}{\sigma_{gr}/\sqrt{n}}\right) \quad (4)$$

For positive seeding factors, if one lets $P(\bar{x} \geq \phi) = \alpha$ and $P(\bar{x}_r < \phi) = \beta$ (or $P(\bar{x}_r \geq \phi) = 1 - \beta$), where α , β are the probabilities of Type I and Type II errors, respectively, then from (3) and (4), it follows that

$$\frac{\phi - \mu_g}{\sigma_g/\sqrt{n}} = z_{1-\alpha} \quad (5)$$

and

$$\frac{\phi - \mu_{gr}}{\sigma_{gr}/\sqrt{n}} = z_\beta \quad (6)$$

where for a two-tail test:

$$\begin{aligned} z_{1-\alpha} &= 2.575 & \text{for } \alpha &= 0.01 \\ &= 1.96 & \text{for } \alpha &= 0.05 \\ &= 1.645 & \text{for } \alpha &= 0.1, \text{ etc.} \end{aligned}$$

From (5) and (6), a formula can be obtained that will give the number of seeded cases as a function of the seeding factor r necessary to test the null hypothesis H_0 : the seeded sample of size n comes from the unseeded population versus the alternative that it does not. From (5), it follows that

$$\phi = z_{1-\alpha} \frac{\sigma_g}{\sqrt{n}} + \mu_g \quad (7)$$

Using (7) and (6), it follows that

$$\sqrt{n} = \frac{z_{1-\alpha}\sigma_g - z_\beta\sigma_{gr}}{\mu_{gr} - \mu_g} \quad (8)$$

At this point we will restrict our test by taking $\alpha = \beta$.

Using the fact that $z_\alpha = -z_{1-\alpha}$:

$$n = \left[\frac{z_{1-\alpha}(\sigma_g + \sigma_{gr})}{\mu_{gr} - \mu_g} \right]^2 \quad (9)$$

In applying (9), the values of $\mu_{gr} - \mu_g$ as a function of seeding factor can be taken directly from Fig. 2 and the values of σ_{gr} can be obtained from Fig. 3, where $\sigma_g =$

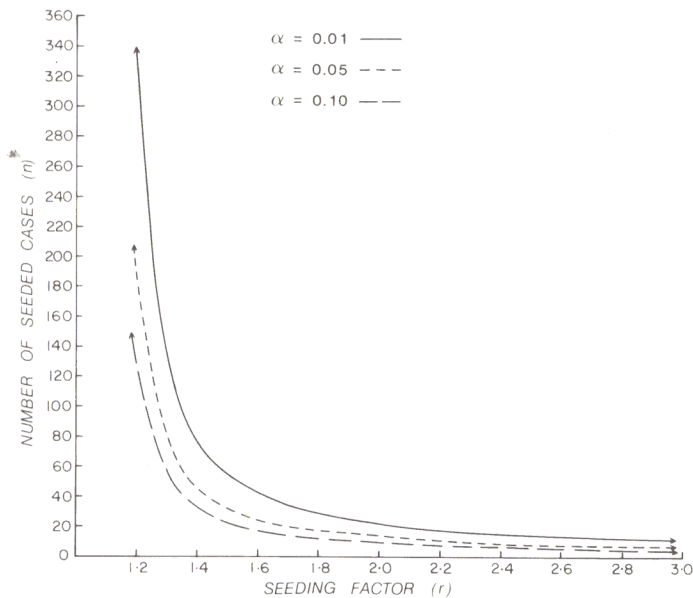


Fig. 5 Number of seeded cases required to detect a seeding factor at various significance levels.

0.37. Figure 5 shows n as a function of r (for $r > 1$) for $\alpha = 0.01, 0.05$ and 0.1 . The arrows indicate that for increasing r , n goes asymptotically to zero and for $r \rightarrow 1$, $n \rightarrow \infty$. If we assume that $\beta > \alpha$ rather than $\alpha = \beta$, then n can be determined from (8). Obviously, in this case n will be smaller. For negative seeding factors \bar{x}_r will be less than \bar{x} , and in this case one should let $P(\bar{x} \geq \phi) = 1 - \alpha$ and $P(\bar{x}_r \geq \phi) = \beta$ (where again α and β are defined as before). Working in a similar manner, it can be found that

$$n = \left[\frac{-z_{1-\alpha}(\sigma_g + \sigma_{gr})}{\mu_{gr} - \mu_g} \right]^2$$

As an example of the above, suppose that one seeded 100 cells and obtained a value of ϕ equal to 0.1. Then from (5) it can be found that H_0 is accepted at $\alpha = 0.01$ and rejected at $\alpha = 0.05$. For $\alpha = 0.05$ and $n = 100$, Fig. 5 shows that $r = 1.28$. For $r = 1.28$, $\mu_{gr} = 0.27$ and $\sigma_g = 0.362$ and (6) shows that one can accept the null hypothesis H_0' : the seeded sample comes from the population with mean $\mu_{gr} = 0.27$ and standard deviation $\sigma_{gr} = 0.362$. Therefore, at $\alpha = 0.05$ the statistically significant seeding factor is 1.28.

Finally, Fig. 5 or the formulas given above can be used to reverse the question and find the significance level at which a certain seeding factor can be detected when n experiments are made. For example, if one seeds 50 cells and expects a seeding factor of about 1.5 the expected significance level in estimating that seeding factor is about 0.03.

b Decaying Cells

Working on the same lines as before, similar results are obtained for the decaying

cells. In this case for $r > 1$:

$$n = \left[\frac{z_{1-\alpha}(\sigma_d + \sigma_{dr})}{\mu_{dr} - \mu_d} \right]^2$$

where $\mu_{dr} - \mu_d$ can be taken from Fig. 2 and σ_{dr} can be obtained from Fig. 3, where $\sigma_d = 0.36$. In general, fewer cells than before are needed. This is to be expected since the variability of the flux in time is somewhat smaller when a cell is decaying than when it is increasing. Thus σ_d and σ_{dr} will be smaller as compared to σ_g and σ_{gr} , and consequently the number of seeded cases will be smaller. For example, for $r = 1.25$ and $\alpha = 0.01$, $n = 144$. In the case of the growing cells $n = 194$.

For $r < 1$, again:

$$n = \left[\frac{-z_{1-\alpha}(\sigma_d + \sigma_{dr})}{\mu_{dr} - \mu_d} \right]^2$$

Before proceeding with the discussion, it should be mentioned that the preceding results are based on the estimated values of μ_g , μ_d , σ_d and σ_g . However, simulations indicate that the results will not be drastically altered if the estimated values are somewhat different. From those simulations, and as illustrated in graphs 2 and 3 if $\mu_g = \mu_g' > 0.12$, then $\sigma_g \rightarrow \sigma_g' < 0.37$. Accordingly, n is somewhat smaller for small seeding effects and practically unaffected for large seeding effects. This can be understood by examining the asymptotic behavior of $\mu_{gr} - \mu_g$, $\mu_{dr} - \mu_d$, $\sigma_d + \sigma_{dr}$ and $\sigma_{gr} + \sigma_g$ in Figs 2 and 3. The reverse will be the case if $\mu_g' < 0.12$. Therefore, even if the parameters of the distribution are somewhat varied, the sample size requirements will be similar to those determined above. In addition, if we had used equations for testing the difference between the means for the seeded and unseeded samples then we would have had:

$$n' = \frac{(z_{1-\alpha} + z_{1-\beta})^2 (\sigma_g^2 + \sigma_{gr}^2)}{\delta^2}$$

where δ is the mean of $D = \bar{x}_r - \bar{x}$. For $\alpha = \beta$ it can be found that n' is approximately twice the order of n as given by (9).

6 Other seeding effects

The results presented up to now were obtained under the assumption that the seeding effect is multiplicative. This assumption has been widely used in the literature, but it is clearly possible that the seeding effect (if any) is, for example, normally distributed or is in fact additive rather than multiplicative. These two possibilities were studied in Tsonis (1982). It was found that the results obtained assuming the seeding factor is distributed normally were similar to those when the seeding effect is multiplicative and equal to the mean of the normal distribution. When the effect was assumed to be additive the results were again of the same order as those obtained by assuming a corresponding multiplicative effect.

7 Comparison with other statistics and some further comments

It is clear that the results seem very promising, as can be judged from the fairly small number of seeded cases required to detect seeding effects. In Tsonis (1982) the

comparison of results obtained with distributions that have a long tail on one or each side shows that a larger seeded sample would be required. In some cases this is as much as 5 times larger. To illustrate the above, a comparison with the lognormal model is presented below. For this purpose, the data and statistic used by Biondini (1976) will be considered. His analysis of 26 unseeded and 26 seeded clouds was based on the statistic $\log R$ where R is the total volume of rainfall resulting from each cloud during its lifetime. It was found that seeding affected the sample mean of the distribution, but did not affect the sample standard deviation, i.e. $s_u = s_s$ where the subscripts u and s stand for unseeded and seeded, respectively. Under lognormality the seeding factor can be expressed as:

$$r = e^{(\bar{x}_s - \bar{x}_u)}$$

where \bar{x}_s , \bar{x}_u are the sample means for the seeded and unseeded cases. Therefore $\bar{x}_s - \bar{x}_u = \log r$.

According to the method developed here, in this case

$$n = \left[\frac{2 z_{1-\alpha} \sigma_u}{\log r} \right]^2$$

where it is assumed that the sample standard deviation is a good estimate of the population's standard deviation. Applying the above formula and taking $\sigma_u = s_u = 1.588$ as reported by Biondini (1976), it can be verified that 4–5 times more experiments are required than when the proposed statistic x is used. It is realized that the data and statistic used in this study are not similar to those used by others, and that comparisons may not be exact. In any case, it seems that the approach presented here is significantly more powerful, apparently because in distributions with long tails the effect of extreme values is more pronounced, and therefore a larger seeded sample will be needed to compensate their effect. Also, as stated above, seeding does not affect the shape of the control distribution when that distribution has one or more tails (Biondini, 1976; Simpson, 1972). That means that $\sigma_{\text{unseeded}} = \sigma_{\text{seeded}}$. In our case σ_{seeded} will be smaller than σ_{unseeded} and it will become smaller as r increases. This will also reduce the number of seeded cases needed (see Eq. (9)).

The method presented here was based on 30-min forecasts (or 30-min flux differences) of isolated rain cells. This choice is arbitrary but serves as a demonstration of the method. It may very well be that statistically significant effects will appear at different time intervals Δt . In any case, the method could be applied for different time intervals. However, for different Δt s the number of seeded cases needed to detect seeding factors will not be the same. In general, if one assumes that (for the growing cells, for example) σ_g , μ_g , σ_{gr} and μ_{gr} are functions of the time interval Δt , then (9) can be expressed as

$$n = \left[\frac{z_{1-\alpha} [\sigma_g(\Delta t) + \sigma_{gr}(\Delta t)]}{\mu_{gr}(\Delta t) - \mu_g(\Delta t)} \right]^2$$

Simulations show that, in general, more seeded cases are required if $\Delta t > 30$ min and less if $\Delta t < 30$ min. This is to be expected since $F_{t+\Delta t} - F_t$ will be larger for $\Delta t >$

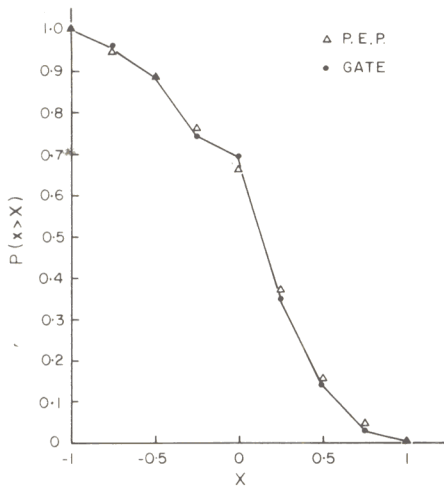


Fig. 6 Cumulative distribution of x for the GATE and PEP data for growing cells.

30 min and smaller for $\Delta t < 30$ min. Therefore, by applying the method for different Δt s, a statement about the time that the seeding needs to act can be made. Moreover, by testing for statistically significant results at different Δt s it may be possible to determine that seeding increased the flux during the early stage and decreased it later, with the final result that over the lifetime of the cell the total amount did not increase. Such results might indicate that seeding accelerates the precipitation process.

In addition, the proposed method may be appropriately applied with rain-gauge data, if the rain-gauge density is sufficient to eliminate the effects of spatial variability. It has been demonstrated (Hildebrand et al., 1979, for example) that with gauge densities ≥ 1 per 100–200 km² mean convective rainfall measurements are quite accurate. With such densities, a control area and a target area could be selected and the proposed method could be straightforwardly adapted to that situation.

Finally, it is possible that the method proposed could be applied to areas where rain is not falling when seeding is performed, as follows: assume a square or rectangular area represents the area that might develop precipitation. From the cross-correlation between two radar patterns separated by a time interval (Tsonis and Austin, 1981), or perhaps from another method, the velocity of the radar pattern can be found. By translating the area considered and applying the proposed statistics for both the control and seeded samples, one can obtain results as previously.

8 Consideration of data from other sites

The results presented above were obtained using the GATE data set. The aim in this section is to examine how data from other sites might alter the results. For this purpose, the data from PEP mentioned in Section 2 were used. Once again, cells were collected (as in the case of GATE) for days 83 and 108 in the 1980 PEP data set. Fifty-six values of the statistic x for the growing cases and fifty-three for the decaying cells were computed. Figure 6 shows the probability that x exceeds the value X plotted as a function of X for the growing cases for GATE and PEP. As can be seen, the cumulative distributions of x at these two places are almost identical. This

similarity may indicate that the qualitative properties of the cells are similar in both places. Therefore, if the distributions of x are the same in both places the same results and conclusions hold for the PEP data. The above statement may not be true for other sites.

9 Conclusions

In this paper a new statistic and technique development are proposed for the design and evaluation of cloud-seeding experiments. The statistic proposed and used here was defined as:

$$x = \frac{F_{t+30} - F_t}{\max(F_{t+30}, F_t)}$$

The essential properties of x are that its distribution is bounded, and that good estimates of the mean and standard deviation of its distribution can be obtained. Because of the boundedness and simple construction of x , it was possible to obtain encouraging results. These results show that statistical tests can be powerful with fairly small numbers of experimental units. For example, a seeding effect of 25% could be detected at significance level $\alpha = 0.1$ with 85 seeded cases. Comparison of the results with those obtained by other methods indicates that the proposed method is very promising. For demonstration purposes only, the formulation of x contains 30-min rain-flux differences. However, the technique can be applied to different Δt s. It is also possible that the method can be extended to non-precipitating clouds that produce rain later in their lifetimes.

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