An Evaluation of Extrapolation Techniques for the Short-Term Prediction of Rain Amounts

A.A. Tsonis and G.L. Austin
Weather Radar Observatory, McGill University
Montréal (Québec)

[Original manuscript received 15 September 1980; in revised form 6 January 1981]

ABSTRACT *Twenty-seven radar cells from the Tropical Atlantic observed during GATE were followed and measurements of their fluxes and areas for initial time increments* T_0 *were fitted to various extrapolation schemes. The extrapolation procedure that gave the smallest error in forecasting the changes in flux and area, was found to be the linear one and the optimum increment* T_0 *was about 30 min. However, even though these techniques have the advantage of establishing a trend in the behaviour of the flux and area with time, a comparison of the forecast errors from the linear extrapolation scheme with those from the "status quo" (persistence) assumption shows little if any improvement.*

A technique including both cell motion and internal changes in flux and area of the rain cells was developed to evaluate the accuracy of rain accumulation forecasts. It was found that the errors generated by the "status quo" assumption were of the order of 77% for a 2-h forecast with little improvement by allowing for the extrapolation of area and flux.

RÉSUMÉ *Vingt-sept cellules tropicales de la zone atlantique ont été observées par radar à l'occasion de ETGA et les mesures de flux et de surface pour diverses périodes initiales* T_0 *ont été appliquées à des méthodes variées d'extrapolation. La meilleure méthode d'extrapolation, soit celle où l'erreur dans la prévision des changements du flux et de la surface est la plus petite, se trouve être la méthode linéaire et l'augmentation optimum du* T_0 *est de 30 min. Toutefois même si ces techniques ont l'avantage d'établir les tendances du comportement du flux et de la surface en fonction du temps, une comparaison entre les erreurs dans les prévisions atmosphériques par extrapolation linéaire et celles de l'hypothese du "statu quo" (persistance) montre très peu d'amélioration.*

Une méthode incluant à la fois les mouvements de la cellule ainsi que les changements internes du flux et de la surface des cellules de pluie a été développée afin d'évaluer la précision des prévisions d'accumulation de pluie. On a pu établir ainsi que les erreurs dues à l'hypothèse du "statu quo" était de l'ordre de 77% pour une prévision de 2 heures avec peu d'amélioration en tenant compte de l'extrapolation de la surface et du flux.

ATMOSPHERE-OCEAN 19 (1) 1981, 54–65 0705-5900/81/0000-0054$01.25/0

1 Introduction

Prediction of the rain amount to be obtained from a shower an hour or so before it arrives at a particular location is a subject that has received little attention in the meteorological literature. This situation occurs even though there are a large number of potential users of such information, ranging from operators of urban sewer systems, small hydroelectric schemes and officials responsible for issuing flash flood warnings. In addition, the ability to predict to a specified accuracy the amount of rain falling from a shower would be a valuable tool in the evaluation of cloud-seeding experiments that are conducted for the purpose of increasing rain in arid areas.

At the McGill Weather Radar Observatory a real-time operation for forecasting precipitation has been going on for four years, (Austin and Bellon, 1974; and Bellon and Austin, 1978). Basically their method is to determine the velocity of each cell from the cross-correlation between two radar patterns that are separated by a certain time interval, usually thirty minutes. Every cell then is assumed to remain constant in area and flux; a forecast is performed by simply translating them with their measured velocities. Other researchers (Greene, 1972; Blackmer and Duda, 1972) developed techniques to track radar echoes for short-time forecasts without taking into account changes in the shape or intensity of the echoes. More recently Huff et al. (1980) allowed for changes in flux and area on the basis of trends established over a period of 10 min. They found that the average error in forecasting areally averaged rainfall at the ground is nearly 100% for a period of about an hour. In a different approach Johnson and Bras (1980) used rain gauge data and a stochastic model to predict the rate of rainfall at a point.

In this paper we will address two separate questions which together constitute the source of error in the rain forecasting scheme. In Section 2 we will discuss extrapolation techniques for time changes in the flux of rain falling out of a particular cell irrespective of its motion, and in Section 3 will examine the impact of these extrapolation schemes on the forecasts of accumulated rain on the ground after taking into account the storm's motion.

2 Extrapolation of time changes of rain flux and area

a *Data Base*

Radar data from the GARP Tropical Atlantic Experiment (GATE) obtained at the Canadian ship *Quadra* were selected for an interesting case of tropical convection. The experiment was conducted in the summer of 1974 about 1500 km off the coast of Senegal. For several days digital maps of the radar-measured precipitation within 200 km of the ship were constructed from the raw tape archive with a temporal resolution of 5 min and a spatial resolution of 4 km × 4 km. The relationship between radar reflectivity (Z) and rainfall (R) used was $Z = 300R^{1.325}$. A computer programme was used to identify, isolate and calculate the flux and area of cells of rain and to print out PPIs in which individual cells are identified by a cell number. Thus, individual cells could be followed in time. The sequence of maps at 5-min intervals was used to select cells or cell complexes. In this study "cell" means any isolated patch of rain appearing as an echo on the radar maps regardless of its size.

Cells were selected by following their history through the map sequence. Each cell was followed from the beginning of its life (i.e. radar life) up to its end, its flux and area being noted at every time step (5 min). Cells that existed when the radar was switched on or that moved into radar range are considered in the sample as well, since these situations represent real occurrences in a radar data set. During their lifetimes cells can merge with other cells or split. It is a matter of definition as to how merging or splitting is regarded. For this study the following arbitrary procedure was adopted:

1) If a "parent" cell splits and the two (or more) "daughter" cells continue, separated by a distance of at least four bins, then the parent ends and the new cells begin their lifetimes.
2) If a parent cell splits and the daughter cells later rejoin, then the parent is assumed to continue its lifetime, ignoring the birth of the daughters.
3) If two (or more) cells exist separately for a reasonably long time, (15 min), and then merge and remain merged, a new cell is considered as starting its lifetime while the two (or more) original cells are considered as having ended theirs.
4) If two cells exist separately, merge and separate then the merging is disregarded. The flux of the cells while merged is found by linearly interpolating the fluxes before and after the merging.

We are only interested in those cells that last for at least 100 min. This is an arbitrary consideration based on the following points: a) since a time interval T_0 is required to establish the extrapolation, the cell should last for a significantly longer time so that a meaningful forecast can be made; b) not many individual cells last much longer than 100 min.

As mentioned above, only long-lived cells are examined, since short-lived cells do not serve any purpose. Twenty-seven cells satisfying the above criteria were selected. The data used were from days 246, 250 and 261. The forecast was extended up to $(100 - T_0)$ min for all cells whether they lasted longer or not, in order to keep the size of the sample constant.

b *Extrapolation Schemes*

The following extrapolation schemes were tested:

i) $\log \bar{F} = a + bt$ (exponential)

ii) $\log \bar{F} = a + b \log t$ (power law)

iii) $\bar{F} = a + bt$ (linear)

iv) $\bar{F} = a + bt + ct^2$ (2nd-degree polynomial)

where $\bar{F}$ denotes the total flux over the area of the cell (mm h^{-1} km^2) and t the time (min).

To test which scheme is best, each cell was considered individually. The flux of the cell is given every 5 min for 100 min. The extrapolation schemes

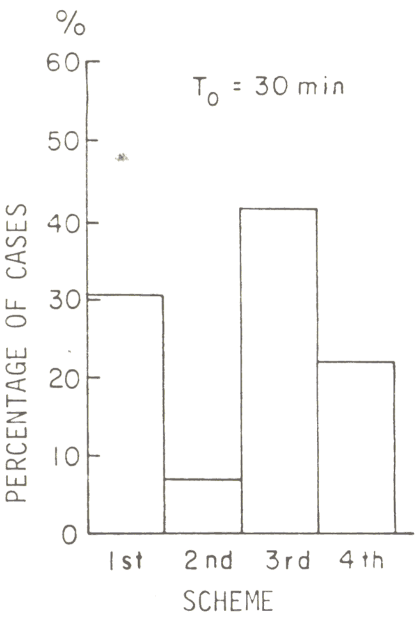

Fig. 1 Percentage of cases at which each extrapolation scheme obtained the smallest value of y.

were fitted to the fluxes for a varying time increment T_0 ($<$ 100 min). After that, extrapolations that provided forecasts for periods increasing by 5-min steps were performed up to a maximum period of 100 − T_0 min. The best extrapolation scheme for each cell was then determined by calculating which one gave the minimum value of y, defined as

$$y = \sum_{i=1}^{n} (\bar{F}_{f_i} - \bar{F}_{0_i})^2$$

where $\bar{F}_f$ is the forecasted total flux, $\bar{F}_0$ the observed total flux and i the number of 5-min intervals.

For $T_0 = 30$ min, Fig. 1 shows the percentage of cases at which each scheme obtained the smallest value of y. It may be seen that most often the linear extrapolation scheme is superior to the others (even though one scheme has an extra degree of freedom). This result was confirmed for other values of T_0.

c *A More Detailed Analysis of the Linear Extrapolation Case*

As noted above, the linear case gave the most promising results, so that a more detailed analysis of this technique is presented below.

Table 1 shows the mean (μ) and the standard deviation (σ) of the percentage error, for different time increments T_0 ranging from 10 to 60 min at each instantaneous forecast time T_i (i.e. at the 5th min, 10th min, ... etc.), where the percentage error (E) is defined as follows:

$$E = \frac{\bar{F}_{f_i} - \bar{F}_{0_i}}{\bar{F}_{0_i}} \times 100$$

TABLE 1. Mean (μ) and standard deviation (σ) of the percentage error as a function of the initial time increment T_0 and the forecast time T_i.

T_i (min)	T_0 (min) 10 μ	10 σ	15 μ	15 σ	20 μ	20 σ	25 μ	25 σ	30 μ	30 σ	35 μ	35 σ	40 μ	40 σ	45 μ	45 σ	50 μ	50 σ	55 μ	55 σ	60 μ	60 σ
5	−4.9	39.0	−9.3	22.9	−3.4	23.7	1.9	17.6	−2.8	*15.5*	−1.3	18.5	2.2	20.7	10.1	35.5	9.4	30.8	9.1	35.2	13.2	35.9
10	−22.6	61.5	−14.9	29.3	−1.6	27.6	−2.6	*21.5*	−2.8	22.6	0.6	22.1	11.1	40.6	12.4	37.7	11.9	40.8	16.1	46.9	9.7	31.4
15	−22.0	76.2	−16.3	38.6	−7.2	32.9	−1.4	29.4	−0.6	*27.8*	8.9	38.5	13.6	43.8	15.4	46.8	20.3	55.5	12.5	40.7	11.0	38.2
20	−25.7	102.0	−24.7	44.4	−6.6	39.6	0.8	*37.8*	6.3	38.0	12.5	47.5	16.5	51.0	25.1	63.8	16.8	48.9	14.3	48.0	11.1	38.1
25	−38.6	128.3	−28.2	53.8	−5.8	53.7	8.6	45.7	9.8	47.1	16.4	60.9	26.0	67.3	21.6	55.4	19.3	58.9	13.8	45.5	7.4	*35.5*
30	−43.8	148.1	−30.1	66.9	5.3	64.9	13.6	59.5	14.0	62.9	27.0	83.1	23.1	61.7	25.1	69.2	18.0	52.7	9.5	*41.8*	12.4	53.1
35	−42.9	192.7	−26.1	87.0	5.1	68.1	16.7	73.7	23.8	81.5	23.0	70.6	25.7	73.7	24.0	64.0	14.0	*50.5*	16.5	63.6	19.1	75.1
40	−32.2	209.6	−26.0	100.3	6.0	91.0	26.9	90.3	20.3	71.8	24.2	76.7	24.9	71.3	20.1	*62.9*	21.3	70.1	24.0	86.0	24.8	107.8
45	−42.5	249.9	−27.5	111.6	16.6	106.0	24.4	86.7	21.3	76.9	23.1	75.1	21.6	*73.1*	28.6	76.5	29.0	90.0	30.5	120.0		
50	−59.5	287.8	−28.4	131.7	13.2	107.2	27.5	102.9	21.1	78.9	18.9	*76.2*	31.2	86.6	42.6	115.5	35.9	121.9				
55	−60.1	326.6	−34.3	147.0	11.4	115.8	27.9	102.7	16.4	*75.5*	28.9	90.9	45.2	120.3	51.4	166.0						
60	−62.1	375.3	−36.7	167.5	7.6	118.6	23.6	104.2	26.7	*94.1*	42.0	120.3	56.1	164.8								
65	−71.7	453.7	−41.0	185.5	7.3	133.6	32.2	118.5	34.8	*110.0*	48.8	146.9										
70	−92.4	498.3	−46.0	214.0	5.8	145.7	43.0	132.0	40.3	*117.7*												
75	−81.4	554.3	−56.6	226.8	25.6	*154.9*	59.4	158.0														
80	−115.7	627.7	−57.9	241.4	39.2	*195.5*																
85	−84.8	627.0	−66.0	*279.8*																		
90	−29.8	768.0																				

In this table the italicized value of σ for each T_i indicates the value of T_0 giving the lowest standard deviation. It can be seen that the optimum value of T_0 is between 25 and 30 min with $T_0 = 30$ min being the value that gives the lowest standard deviation most often.

Cearly, if T_0 is too short the trend will not be well defined. On the other hand if T_0 is too long, the possibility of the cell having undergone a significant change in character (e.g. from increasing to decreasing rain flux), becomes more likely. From Table 1 it can be seen that both the mean and the standard deviation of the percentage error increase with increasing T_i, and for $T_0 = 30$ min, for a one-hour forecast the error reaches a mean value of about 25% with a standard deviation of about 95%. Here, it must be mentioned that these errors refer only to the fluxes deduced from the radar data and that there may be an additional error arising from discrepancies between radar measured rain-flux and the actual rain-flux.

d *Extrapolation of Rain Area*

A similar analysis was carried out, using the same data base, for the area of the raining cells. The results were almost identical to those for the extrapolation of fluxes in that the linear scheme worked best with optimum time increment T_0 equal to 30 min. This result is consistent with the finding that the total flux ($\bar{F}$) of rain is linearly related to the storm area (A), i.e. $\bar{F} = CA$ where C is the mean rainfall rate for raining areas (Lovejoy and Austin, 1979). This result was confirmed for GATE convective data and for convective and stratiform rain over the Montreal area.

The last point that was investigated was to see whether or not the linear extrapolation technique provides an improved forecast. For this purpose the results obtained by linear extrapolation were compared with results obtained from the assumption that the flux for every T_i remains constant and equals that value of the flux for the last observation in the time increment T_0. Figs 2 and 3 show the comparisons of the mean and standard deviation for $T_0 = 30$ min. No significant difference can be seen. Whether or not a trend is established the errors in the forecast are very similar: a conclusion found to hold for all T_0 values.

The flux characteristically fluctuates significantly from one time step to another. Therefore, the trend may often be incorrectly defined from the information in the time increment T_0, so that on average any extrapolation technique does not give a better result than that arising from the "status quo" assumption. Furthermore, in cases where the cell initially grows and subsequently decays, it is obvious that any linear extrapolation will be unsuccessful.

3 Forecasting accumulative rain amount

The additional step required to allow the forecasting of rain amounts on the ground is to translate the cell at its measured horizontal velocity. Thus the steps taken in this analysis are:

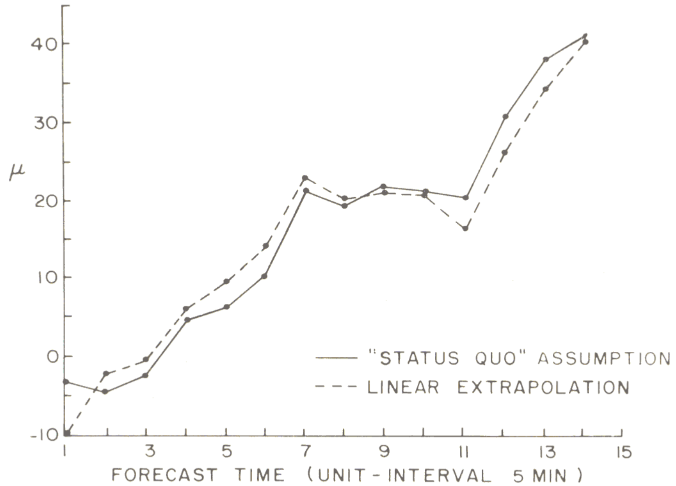

Fig. 2 Mean of the percentage error as a function of the forecast time ($T_0 = 30$ min).

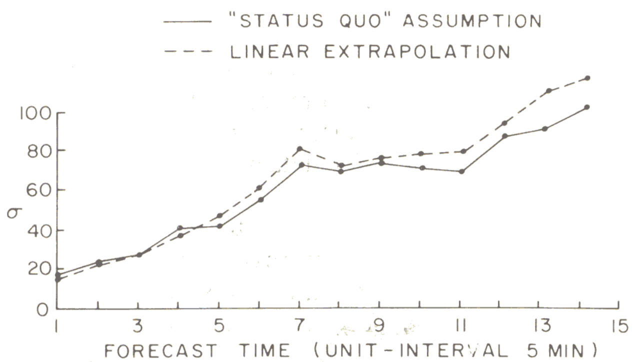

Fig. 3 Standard deviation of the percentage error as a function of the forecast time ($T_0 = 30$ min).

a) Measurement and extrapolation of the motion.
b) Forecasting changes in flux and area.
c) Accumulation of the rain at a point.

a *Motion Forecasting*

The method used to find the motion of the cell is the one used in the real-time operation of SHARP, (Short Automated Radar Procedure), (Austin and Bellon, 1974). This technique is based on finding the cross-correlation between two radar patterns separated by Δt (= 30 min). Mathematically this is expressed as:

$$\gamma(x_0, y_0, \Delta t) = \frac{1}{N\sigma_i\sigma_{i+1}} \int\int [B(x, y, t_i) - \overline{B(t_i)}] \times [B(x + x_0, y + y_0, t_{i+1}) - \overline{B(t_{i+1})}]\, dxdy \quad (1)$$

where $\gamma(x_0, y_0, \Delta t)$ is the cross-correlation coefficient between two patterns separated by a time interval $\Delta t = t_{i+1} - t_i$ and by a spatial lag given by (x_0, y_0).

$B(x, y, t_i)$ is the spatial distribution of pattern at time t_i – in our case, the pattern is a precipitation echo. With ordinary radar reflectivity weighting, B is quantized in integer values such that $0 \leq B \leq 15$.

$N = \iint dxdy$ is the area of integration which must contain both patterns, $\overline{B(t_i)}$ is the average value of field B over the area N at time t_i, i.e.

$$\bar{B} = \iint B(x, y, t_i) dxdy/N.$$

The standard deviation of B over N at time t_i is

$$\sigma_i = \{\iint [B(x, y, t_i) - \overline{B(t_i)}]^2 \, dxdy/\mathrm{N}\}^{\frac{1}{2}}.$$

For the maximum correlation,

$$\gamma_{\max}(a_0, b_0, \Delta t) = \frac{1}{N\sigma_i\sigma_{i+1}} \int\int [B(x, y, t_i) - \overline{B(t_i)}] \times [B(x + a_0, y + b_0, t_{i+1}) - \overline{B(t_{i+1})}] \, dxdy, \quad (2)$$

which incorporates changes in the outer boundary and internal structure of the cell; a_0 and b_0 yield the best match between the two patterns. From a_0 and b_0 the motion can be found as follows:

$$\mathbf{V} = (u^2 + v^2)^{\frac{1}{2}}(\mathbf{u}', \mathbf{v}') \quad (3)$$

where $u = a_0/\Delta t$, $v = b_0/\Delta t$ and $(\mathbf{u}', \mathbf{v}')$ is the unit vector in the direction of a_0 and b_0.

b *Changes in Flux and Area*

During the time increment T_0 the changes in area and flux as a function of time are considered and a linear extrapolation is applied to obtain the forecast values (see Section 2b). The results are applied in the following manner.

If the extrapolation procedure predicts an increase in the size of the cell, the increased area is added at random points around its boundary. A computer programme first finds the coordinates of the boundary of the cell and then randomly chooses points where increases of the area are to be distributed at the forecast time. For the data set used, each point corresponds to an area of 16 km^2 (spatial resolution 4 km × 4 km), and the number of points chosen depends on the increase of the area forecast by the linear extrapolation. Figure 4 shows a typical example of a cell with an area of 8416 km^2 that has been increased by 5.7% – this increase is indicated by the circles. The numbers are rainfall rate on a logarithmic scale. In these new areas of rain the rainfall rate is selected to lie randomly between 0.1 to 1 mm h^{-1}. According to the logarithmic scale of rainfall rate this range corresponds to the number 1. This means that in Fig. 4 for each circled point the number 1 is entered. The internal changes of the flux at each point (i, j) are considered according to the algorithm:

$$F_{i,j,T_0+\Delta t} = F_{i,j,T_0} + aF_{i,j,T_0} \quad (4)$$

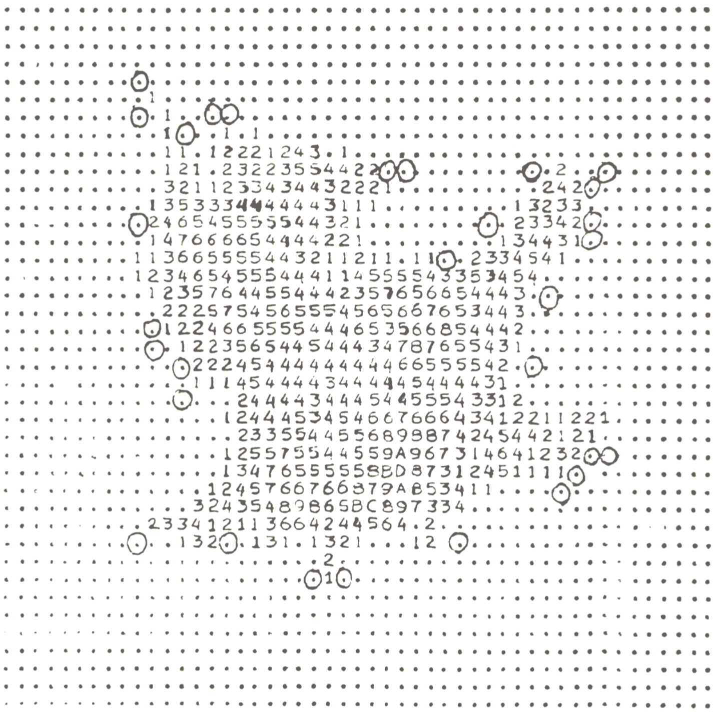

Fig. 4 Digital radar map of a cell increased in area by 480 km² using the scheme described in the text.

where a is defined as the rate of increase:

$$a = \frac{\bar{F}_{T_0+\Delta t} - \bar{F}_{T_0} - R}{\bar{F}_{T_0}} \tag{5}$$

and $\bar{F}_{T_0+\Delta t}$, $\bar{F}_{T_0}$ are the total fluxes of the cell at $T_0 + \Delta t$ (forecast) and T_0, respectively, and R is the flux resulting from the increase in the area.

In the case where the cell is forecast to decrease in size the model is essentially the same except that the cell area is decreased by setting the rainfall equal to zero at randomly chosen points around the boundary. For the decreasing cell case the coefficient a in (4) is defined as:

$$a = \frac{\bar{F}_{T_0+\Delta t} - \bar{F}_{T_0} + R}{\bar{F}_{T_0}}$$

where R is the resulting decrease of the flux arising from the decrease of the area.

c *Accumulation at a Point*

Accumulation at a point was accomplished by using a computer subroutine to process data from a series of real or forecast maps. This procedure is similar to that used in the real time operation of SHARP.

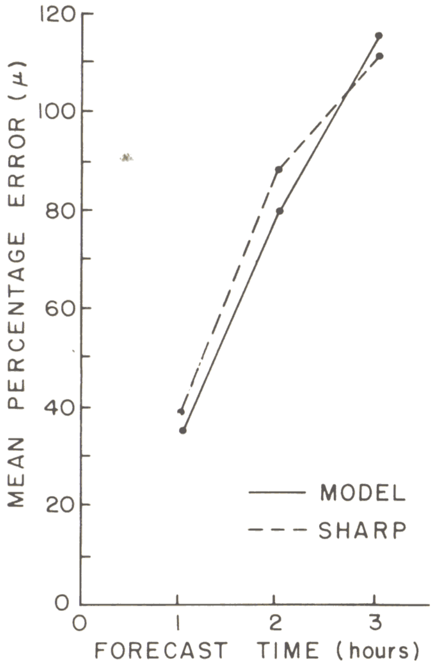

Fig. 5 Mean of the percentage error of the accumulated amount of rain at a point as a function of the forecast time.

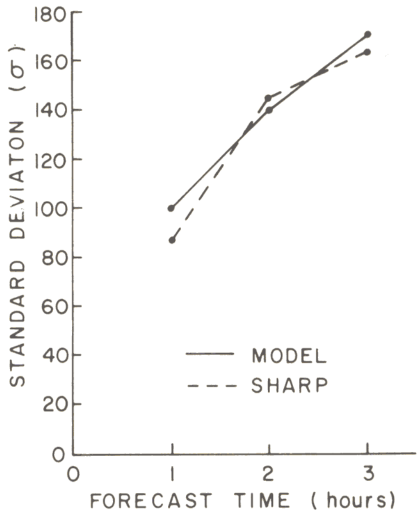

Fig. 6 Standard deviation of the percentage error of the accumulated amount of rain at a point as a function of the forecast time.

d *Data Analysis*

According to the model, merging or splitting of the cells could be predicted. This, however, leads to a problem in verification since the cells are followed individually and the area and flux of each cell are forecast separately. When two or more cells merge, information for the new "merged" cell is difficult to interpret so that these cases were eliminated from the data base considered. On occasion, it was observed that one of the two variables (flux and area) is forecast to be zero before the other. It was arbitarily decided that if the flux or area was forecast to be zero then both would be set equal to zero.

Eleven storms were selected over the Montreal region to test the model using data for 27, 28 and 29 June 1978. The Montreal data were chosen in preference to GATE data for this part of the study, since we wished to compare the results with those of the operational procedure mentioned in the Introduction.

e *Results*

Figures 5 and 6 show the mean percentage error of the accumulated amount of rain at a point (μ) and its standard deviation (σ) for 1-, 2- and 3-h forecasts. In the same figures the corresponding results using the "status quo" assumption (SHARP) are shown. Errors are due to two factors: errors in forecasting the flux and the area; and errors in translating the cell.

Since the motion is calculated in the same way in both methods the extrapolation techniques can be compared directly. Whether or not internal changes of the flux or area are considered, an insignificant difference of the order of 5–7% results in the mean percentage error of the forecast.

4 Conclusions

In this study extrapolation techniques were applied in order to forecast the total flux (or area) of a cell. Of those techniques tested, linear extrapolation was found to give the smallest errors. The mean and the standard deviation of the percentage errors increase with the length of the forecast period. The duration of data that was used to fit the extrapolation schemes was varied; and an interval of 30 min was found to be the most appropriate. It was suggested that this is due to two opposing factors, one acting in establishing the trend and the other in increasing the possibility of changes in the character of the cell. Comparison of the results with those obtained by the ''status quo'' assumption shows that these types of extrapolation techniques really do not yield significant forecast improvements. It was felt that this could be due to either the failure of the method to follow changes in the flux when growing cells start to decay or the variability of the flux from one time step to another dominating a weakly defined trend.

However, it must be mentioned that preliminary work using the stochastic methods of Box and Jenkins (1970) for forecasting the total flux and area of a cell showed no advantages. They did not, in general, yield smaller errors in the forecast, and furthermore they also have the disadvantage that a long time increment T_0 is needed to perform the extrapolation.

Attempts to forecast rain accumulation at the ground have yielded generally similar results. The errors in mean rain amount for a two-hour forecast for example are of the order 70%, with the ''status quo'' assumption adding only another 5–7%. However, the results from this study were obtained from long-lived cells and it may be that they are not valid for short-lived cells.

Results of Huff et al. (1980) give even higher errors in this type of forecast scheme, but that, we suspect, is due to the very short time increment T_0 used to establish the trends (10 min).

Acknowledgements

Ms A. Kilambi, Drs S. Radhakant and A. Bellon are thanked for their assistance in data processing and S. Lovejoy for his helpful discussion. Dr H. Leighton's assistance in this work is greatly appreciated.

References

AUSTIN, G.L. and A. BELLON. 1974. The use of digital weather radar records for short-term precipitation forecasting. *Q.J.R. Meteorol. Soc.* **100**: 658–664.

BELLON, A. and G.L. AUSTIN. 1978. The evaluation of two years of real-time operation of a short-term precipitation forecasting procedure (SHARP). *J. Appl. Meteorol.* **17**: 1778–1787.

BLACKMER, R.H., JR. and R.O. DUDA. 1972.

Application of pattern recognition techniques to digitized radar data. Preprints 15th Radar Meteorology Conference, 10–12 Oct., Champaign – Urbana, Ill., 138–143.

BOX, G.E.P. and G.M. JENKINS. 1970. *Time Series Analysis; Forecasting and Control*. Holden-Day, San Francisco, 553 pp.

GREENE, D.R. 1972. A comparison of echo predictability: constant elevation vs. VIL radar-data patterns. Preprints 15th Radar Meteorology Conference, 10–12 Oct., Champaign – Urbana, Ill., 111–116.

HUFF, F.A.; S.A. CHANGNON and J.L. VOGEL. 1980. Convective rain monitoring and forecasting system for an urban area. Preprints, 19th Conference on Radar Meteorology, 15–18 April, Miami Beach, Fla., 56–61.

JOHNSON, E.R. and R.L. BRAS. 1980. Multivariate short-term rainfall prediction. *Water Resour. Res.* **16**: 173–185.

LOVEJOY, S. and G.L. AUSTIN. 1979. The sources of error in rain amount estimating schemes from GOES visible and IR satellite data. *Mon. Weather Rev.* **107**: 1048–1054.